Factoring Trinomials Using the Key Number Method

(http://www.sheboygan.uwc.edu/developmental-math/BAW/thirteen/lesson13.htm)

A trinomial is a polynomial with exactly three terms. These polynomials have a very special form since they are the typical polynomials that come out of the FOIL method for multiplying two binomials. The Key Number Method of factoring applies to any trinomials $ax^2 + bx + c$, where $a$, $b$, and $c$ are integers and $x$ represents any letter variable or string of variables.

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<th><strong>Key Number Method</strong></th>
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<td><strong>To Factor</strong> $ax^2 + bx + c$</td>
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**Example 1:** Factor $6x^2 + x - 15$.

Find the key number: $(6)(-15) = -90$.

Find factors of -90 that add up to 1 (the middle term is 1$x$). Since $90 = 1 \cdot 90$ or $2 \cdot 45$ or $3 \cdot 30$ or $5 \cdot 18$ or $6 \cdot 15$ or $9 \cdot 10$, it looks like $+10$ and $-9$ will work since they multiply to -90 and add to +1.

Substitute two terms for the middle term whose coefficients equal the factors found in step 2.

$6x^2 + x - 15 = 6x^2 + 10x - 9x - 15$

Factor by grouping.

$6x^2 + 10x - 9x - 15$

$= (6x^2 + 10x) + (-9x - 15)$

$= 2x(3x + 5) - 3(3x + 5)$

$= (3x + 5)(2x - 3)$

$= (3x + 5)(2x - 3)$

$= 6x^2 - 9x + 10x - 15$

$= 6x^2 + x - 15$

**Step 1:** Calculate the product of the first and last coefficients: $a \cdot c$. This is called the key number.

**Step 2:** Find two factors of the key number $ac$ whose sum is $b$ (the middle coefficient).

**Step 3:** Rewrite the original trinomial as a four term polynomial: replace the middle term by two terms that have coefficients equal to the factors found in step 2.

**Step 4:** Factor the four term polynomial by grouping.

**Step 5:** Check by multiplying (use FOIL).