Math 1301 - Test #2 **Name:** Answer Sheet - Tanu

Date: 4-11-13 **Spring 2013** **Student ID:**

Show work where work is required. This can be worth partial credit.

1a. (+3) A line passes through the points $(\overset{x_1}{-4}, \overset{y_1}{-10})$ and $(\overset{x_2}{5}, \overset{y_2}{8})$. Find the slope of the line. Show your work.

slope = $\boxed{2}$ Slope: $\frac{y_2-y_1}{x_2-x_1} = \frac{8-(-10)}{5-(-4)} = \frac{18}{9} = 2$ ✓

1b. (+3) Write the formula for the line described in part a.

$y = \boxed{2(x-5)+8}$ ✓ $y = m(x-x_1)+y_1$

$y = 2(x+4) - 10 = 2x-2$

2. (+6) The weekly sales y of Bruno's Sneakers (measured in thousands of pairs) are a linear function of the price per pair x measured in dollars. When the price is \$60 the weekly sales are 4 thousand pairs. But when the price is \$90 the weekly sales are only 2 thousand pairs. $y = -1/15(x-90)+2 = -\frac{1}{15}x+8$

2a. Find the formula for this linear function. Show your work. $y = \boxed{\frac{-1}{15}(x-60)+4}$ ✓

x = dollars
y = weekly sales (in thousands)

(60, 4)
(90, 2)

Slope: $\frac{y_2-y_1}{x_2-x_1} = \frac{2-4}{90-60} = \frac{-2}{30} = \frac{-1}{15}$

2b. Predict the weekly sales at a price of \$15. *weekly sales* = $\boxed{7 \text{ thousand}}$ ✓

$y = \frac{-1}{15}(x-60)+4$
$= \frac{-1}{15}(15-60)+4$
$= \frac{-1}{15}(-45)+4$
$= \frac{45}{15}+4$
$= 3+4$

3. (+6) According to a certain architect, the amount of cooling needed to air-condition an area is directly proportional to the size of the area. Suppose $\overset{y}{5,000}$ BTUs of cooling are needed to air-condition an area of $\underset{x}{250}$ square feet.

3a. Find the constant of proportionality. Show your work. $k = \boxed{20}$ ✓

$y = k \cdot x$ $\frac{5000}{250} = \frac{k \cdot \cancel{250}}{\cancel{250}}$ $20 = k$

3b. Write the formula for the function.

$y = \boxed{20 \cdot x}$ ✓

3c. How much cooling is required for an area of 370 square feet? Show your work. $\boxed{7,400}$ BTUs ✓

$y = 20 \cdot x$
$= 20(370)$
$= 7400$

4. (+6) The additional baggage fee, y, charged by Global Airlines is a function of the weight, x, of the baggage. This function can be represented by the following piece-wise linear formula:

$$y = f(x) = \begin{cases} 20 & , \text{if } 0 < x \leq 25 \\ 2x - 30 & , \text{if } 25 < x \leq 50 \\ 4x - 130 & , \text{if } x > 50 \end{cases}$$

4a. What is fee for baggage weighing 30 lb? Express this information using function notation.

$f(\underline{30}) = \underline{30}$ ✓

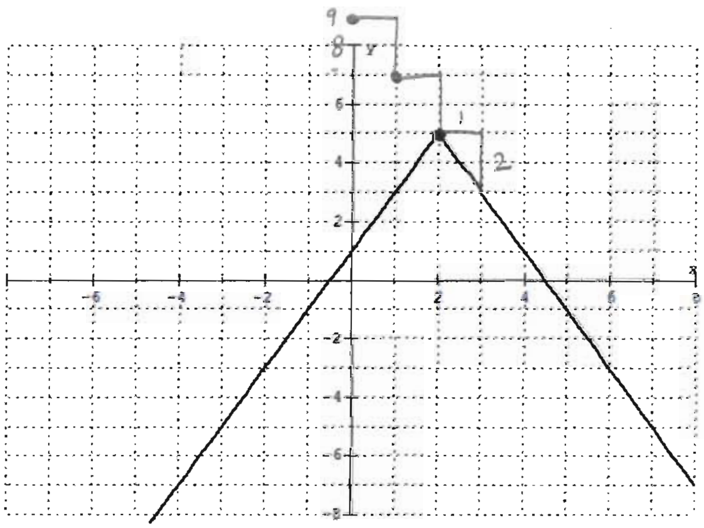
$y = 2(30) - 30$
$= 60 - 30$

4b. Evaluate $f(60)$ and then interpret this value in the context of the problem.

$f(60) = 4(60) - 130$
$= 110$

There is a $110 fee for baggage weighing 60 lbs. ✓

5. (+6) Complete the formula for the piece-wise linear function f whose graph is shown here.



$$f(x) = \begin{cases} \boxed{2x + 1} & , \text{if } x \leq \boxed{2} \\ \boxed{-2x + 9} & , \text{if } x > \boxed{2} \end{cases}$$ ✓

6. (+4) Find the x-intercept of the function $y = \frac{1}{4}x - \frac{3}{8}$. Show your work.

$\boxed{x = 3/2}$ ✓

x int → set y=0

$0 = \frac{1}{4}x - \frac{3}{8}$

$+\frac{3}{8} \quad + 3/8$

$(4)\frac{3}{8} = \frac{1}{\cancel{4}}x(\cancel{4})$

$\frac{3}{2}\ \frac{6}{\cancel{4}}\ \frac{\cancel{12}}{\cancel{8}} = x$

7. (+9) Solve each of the following equations. Give exact solutions, not decimal approximations. Show your work.

7a. $5(x-3)-1=12x+5$

$x =$ $\boxed{-3}$

$5(x-3)-1=12x+5$

$5x-15-1=12x+5$

$5x-16=12x+5$

$-5x \qquad -5x$

$-16=7x+5$

$-5 \qquad -5$

$\frac{-21}{7}=\frac{\cancel{7}x}{\cancel{7}}$

$-3=x$

7b. $\dfrac{2x+2}{8}+\dfrac{x-3}{2}=\dfrac{7}{4}$

$x =$ $\boxed{4}$

$\left[\frac{2x+2}{8}+\frac{x-3}{2}=\frac{7}{4}\right]\times 8$

$2x+2+4(x-3)=14$

$2x+2+4x-12=14$

$6x-10=14$

$+10 \qquad +10$

$\frac{\cancel{6}x}{\cancel{6}}=\frac{24}{6}$

$x=4$

7c. $|4-2x|=20$

$x =$ $\boxed{-8, 12}$

$4-2x=20$

$-4 \qquad -4$

$\frac{\cancel{-2}x}{\cancel{-2}}=\frac{16}{-2}$

$x=-8$

$4-2x=-20$

$-4 \qquad -4$

$\frac{\cancel{-2}x}{\cancel{-2}}=\frac{-24}{-2}$

$x=12$

8. (+6) Solve each of the following inequalities. Write your answers using interval notation. Show your work.

8a. $7-(3-x)\geq 5x-20$

solution set = $\boxed{(-\infty, 6]}$

$7-3+x\geq 5x-20$

$4+x\geq 5x-20$

$-x \qquad -x$

$4\geq 4x-20$

$+20 \qquad +20$

$\frac{24}{4}\geq\frac{\cancel{4}x}{\cancel{4}}$

$6\geq x$

8b. $22<5x+7<42$

solution set = $\boxed{(3, 7)}$

$22<5x+7<42$

$-7 \qquad -7 \qquad -7$

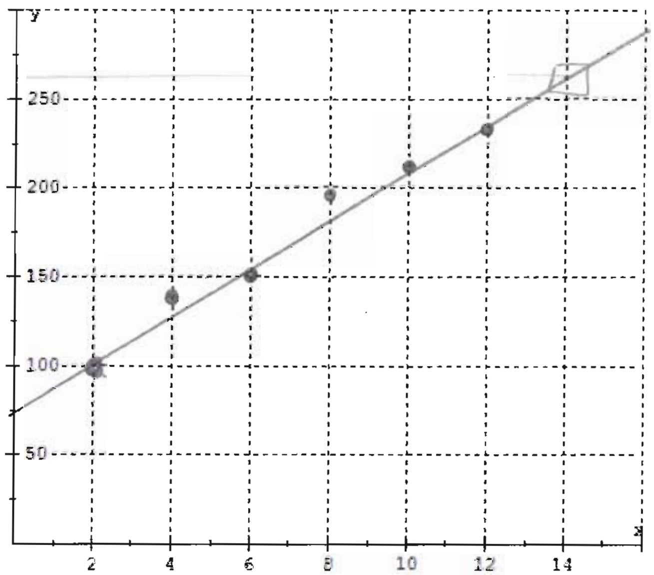
$\frac{15}{5}<\frac{\cancel{5}x}{\cancel{5}}<\frac{35}{5}$

$3<x<7$

9. (+9) The data contained in the following table shows the number of English majors at Deerland State University several years. The variable y represents the number of English majors, and x stands for the elapsed number of years since 2000.

x	2	4	6	8	10	12
y	100	135	150	195	210	230

9a. Make a plot of the data on the set of axes shown here.



9b. Draw a line by hand that you feel "fits" or "represents" this set of data. Make sure the line extends all the way across the graphing window. Answers may vary.

9c. Use the line drawn in part 9b to estimate the number of English majors in 2014.

$\boxed{260}$ ✓

10. (+5) The daily sales y of a particular item at the Deerland State University bookstore is given by the following formula, where x is the price of the item.

$$\text{sales} \rightarrow y = 60 - \frac{3}{2}x \leftarrow \text{price}$$

What is the x-intercept of this function? x-int. $=$ $\boxed{40}$ ✓ Interpret this value in the context of the problem.

set $y=0$

$$0 = 60 - \frac{3}{2}x$$
$$+\frac{3}{2}x \qquad +\frac{3}{2}x$$
$$\frac{2}{3} \cdot \frac{3}{2}x = 60 \cdot \frac{2}{3}$$
$$x = \frac{120}{3}$$
$$= 40$$

When the price of an item is \$40, there are no sales for that item.

11. (+6) The quantity y is $\boxed{\text{inversely}}$ proportional to the quantity x, and $y = 30$ when $x = 6$.

11a. Find the constant of proportionality. $k = \boxed{180}$ ✓

$y = \frac{k}{x}$ $6 \cdot 30 = \frac{k}{6} \cdot 6$

$180 = k$

11b. Write the formula for the function.

$y = \boxed{180/x}$ ✓

11c. Find the value of y when $x = 30$. $y = \boxed{6}$ ✓

$y = \frac{180}{x}$

$= \frac{180}{30}$

$= 6$

12a. (+4) A line passes through the point $(5, 8)$ and is parallel to the line $3x - 9y = 5$. Find the slope of the line. Show your work.

slope $= \boxed{1/3}$ ✓

Parallel; same slope

$3x - 9y = 5$
$-3x \quad -3x$

$\frac{-9y}{-9} = \frac{-3x}{-9} + \frac{5}{-9}$

$y = \frac{1}{3}x - \frac{5}{9}$

12b. (+4) Write the formula for the line described in part a.

$y = \boxed{\frac{1}{3}(x-5)+8}$ ✓

$= \frac{1}{3}x + 19/3$

$y = m(x - x_1) + y_1$

$= \frac{1}{3}(x-5)+8$

13. (+8) The monthly profit P for Giddy Kitty cat food is given by

$$P = \overset{b}{10x} - \overset{a}{x^2} - \overset{c}{21}$$

where x represents the monthly sales in thousands of pounds, and the monthly profit is measured in thousands of dollars.

13a. What monthly sales amount will lead to the maximum profit? What is the maximum monthly profit? Show your work.

↳ vertex

sales amount $= \boxed{5{,}000}$ ✓ pounds

maximum profit $= \boxed{4{,}000}$ ✓ dollars

$x = \frac{-b}{2a}$ $y = f(\frac{-b}{2a})$

$= \frac{-10}{2(-1)}$ $= 10(5) - (5)^2 - 21$

$= \frac{-10}{-2}$ $= 50 - 25 - 21$

$= 5$ $= 4$

13b. What are the two break-even sales amounts for Giddy Kitty (i.e. the sales amounts that result in no profit or loss)? Show your work.

sales amount $= y = 0$

$\boxed{3{,}000}$ pounds

$\boxed{7{,}000}$ pounds ✓

$0 = 10x - x^2 - 21$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-10 \pm \sqrt{10^2 - 4(-1)(-21)}}{2(-1)} = \frac{-10 \pm \sqrt{100 - 84}}{-2} = \frac{-10 \pm \sqrt{16}}{-2} = \boxed{\frac{-10 \pm 4}{-2}}$

$\frac{-10+4}{-2} = 3$ $\frac{-10-4}{-2} = 7$

14. (+15) Consider the quadratic function with the formula $y = f(x) = x^2 - 6x + 3$. Show your work for the following questions.

14a. Fill in the blanks: The graph of f is a ___parabola___ that opens ___upward___.

+3 Its coefficients are $a =$ ___1___, $b =$ ___-6___, and $c =$ ___3___.

14b. Find the y-intercept of f. y-int. $= \boxed{3}$ +2

14c. Find the coordinates of the vertex of f. vertex $= \boxed{(3,-6)}$ +4

$x = \frac{-b}{2a} = \frac{-(-6)}{2(1)} = \frac{6}{2} = 3$

$y = f(\frac{-b}{2a}) = 3^2 - 6(3) + 3$
$= 9 - 18 + 3$
$= -6$

+4 **14d.** Find the x-intercept(s) of f. First give exact answers without decimals, and then give answers rounded to one decimal place.

$3+\sqrt{6} \approx 5.4$	$3-\sqrt{6} \approx 0.6$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
$= \frac{6 \pm \sqrt{(-6)^2 - 4(1)(3)}}{2(1)}$
$= \frac{6 \pm \sqrt{36-12}}{2}$
$= \frac{6 \pm \sqrt{24}}{2}$
$= \frac{6 \pm \sqrt{4 \cdot 6}}{2}$
$= \frac{6 \pm 2\sqrt{6}}{2}$
$= \frac{\cancel{2}(3 \pm \sqrt{6})}{\cancel{2}}$
$= 3+\sqrt{6},\ 3-\sqrt{6}$

14e. Use your answers from parts 14a-14d to sketch a graph of the function f on the set of axes shown here.

+2