Math 1301 - Test #2 Sample Name: Answer Sheet

Date: _____ **Fall 2012** **Student ID:** 11-12-12

Show sufficient work and intermediate results to justify final results or answers. This may be worth partial credit. Neatness counts. You may find the following formulas helpful:

$$m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1} \qquad y = m(x - x_1) + y_1 \qquad y = mx + b \qquad x = \frac{-b}{2a} \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1. Find the x-intercept(s) of the function $3x + 7y = 4$.

4/3

$3x + 7 \cdot 0 = 4$

$3x = 4$

$x = 4/3$

2. Find the x-intercept(s) of the function $y = \frac{2}{3}x - \frac{7}{3}$.

7/2

$0 = \frac{2}{3}x - 7/3 \rightarrow \frac{3}{2} \cdot \frac{7}{3} = \frac{2}{3}x \cdot \frac{3}{2}$

$7/3 = 2/3\,x \rightarrow 7/2 = x$

3. Solve each of the following equations. Give exact solutions, not decimal approximations.

3a. $12 - (3 - 2x) = 5(x - 2)$

$x =$ 19/3

$12 - 3 + 2x = 5x - 10$

$9 + 2x = 5x - 10$

$9 + 10 = 5x - 2x \rightarrow 19 = 3x \rightarrow \frac{19}{3} = x$

3b. $5(x - 2) = 2(x + 6) + 3x$

$x =$ no solution

$5x - 10 = 2x + 12 + 3x$

$5x - 10 = 5x + 12 \rightarrow 5x - 5x = 12 + 10$

$0 = 22$, false

3c. $\frac{5x - 1}{12} - \frac{1}{8} = 2 + \frac{x}{3}$

$x =$ 53/2

LCD = 24

$\frac{24(5x-1)}{12} - \frac{24 \cdot 1}{8} = 24 \cdot 2 + \frac{24x}{3}$

$2(5x - 1) - 3 = 48 + 8x \rightarrow 10x - 2 - 3 = 48 + 8x$

$10x - 8x = 48 + 2 + 3$

$2x = 53$

$x = 53/2$

3d. $|5x + 2| = 7$

$x =$ 1, -9/5

$5x + 2 = 7$ or $5x + 2 = -7$

$5x = 5$ $5x = -7 - 2$

$x = 1$ $5x = -9 \rightarrow x = -9/5$

4. Solve each of the following inequalities. Write your answers using interval notation.

4a. $12 - (3 - 2x) > 5(x - 2)$

solution set = $(-\infty, 19/3)$

$12-3+2x > 5x-10$

$9+2x > 5x-10$

$2x-5x > -10-9$

$-3x > -19$

$x < 19/3$

4b. $4 \leq 5 - 3x \leq 11$

solution set = $[-2, 1/3]$

$4-5 \leq -5+5-3x \leq 11-5$

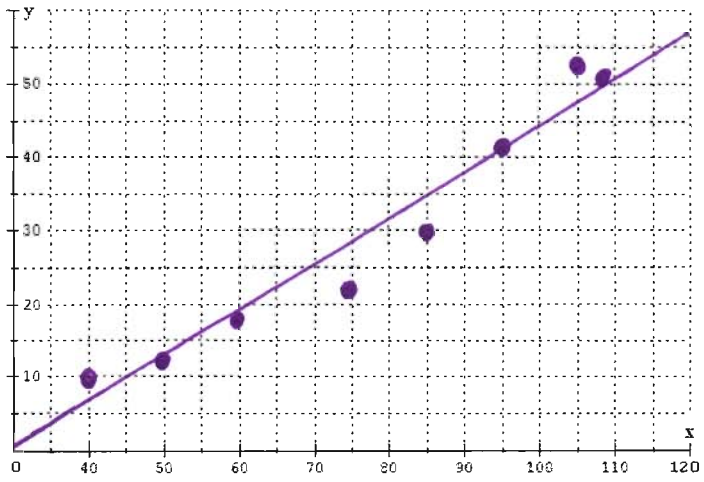
$-1 \leq -3x \leq 6$

$1/3 \geq x \geq -2$

5. Use the following table to answer problems 5a - 5c. This table shows the average amount of chicken consumed by an American yearly.

x (elapsed years from 1900)	40	50	60	75	85	95	105	108
y (chicken consumption in lb)	10	12	18	22	30	41	53	51

5a. Make a scatter plot of this data on the set of axes shown here.



5b. Draw a straight line that "fits" your scatter plot from part 5a. Make sure the line extends all the way through the set of data points. Answers may vary.

5c. Use the line drawn in part 5b to predict the consumption in 2020. Answers may vary. 57 lb

6a. Find the slope of the line containing the points $(-3, -2)$ and $(7, 4)$. Show your work.

slope = 3/5

$m = \frac{4-(-2)}{7-(-3)} = \frac{6}{10} = \frac{3}{5}$

6b. Find the equation of the line containing the points $(-3, -2)$ and $(7, 4)$. Show your work.

$y =$

$y-(-2) = \frac{3}{5}(x-(-3))$

$y+2 = \frac{3}{5}(x+3)$

$y = \frac{3}{5}(x+3)-2$

7a. Find the slope of the line containing the points $(-3, -2)$ and $(3, -2)$. Show your work.

slope = 0

$m = \frac{-2-(-2)}{3-(-3)} = \frac{0}{6} = 0$

7b. Find the equation of the line containing the points $(-3, -2)$ and $(3, -2)$. Show your work.

$y =$ -2

$y-(-2) = 0(x-(-3))$

$y+2 = 0$

$y = -2$

8. Due to urbanization, the deer population in Dry Gulch County has been steadily decreasing since 1970. Scientists have determined the following linear function that relates the number of deer (d) to the elapsed time in years since 1970 (t).

$$d = 13\,000 - 250t$$

What is the x-intercept of this function? x-int. = 52 Interpret this value in the context of the problem.

$0 = 13000 - 250t$

$250t = 13000$

$t = 52$

The deer population will be extinct in 2022.

9. The quantity y is directly proportional to the quantity x, and $y = 15$ when $x = 3$. Find the constant of proportionality k.

$k =$ 5

$y = kx$

$15 = k \cdot 3$

$5 = \frac{15}{3} = k$

10. The number of duty officers (y) needed to guard the Dry Gulch County Jail is directly proportional to the current number of inmates (x). When there are 36 inmates, then 8 duty officers are needed.

10a. Find the constant of proportionality. $k =$ $\boxed{2/9}$

$y = kx$

$8 = k \cdot 36$

$\frac{2}{9} = \frac{8}{36} = k$

10b. Write the formula for the function.

$y =$ $\boxed{2/9\,x}$

10c. How many duty officers are needed when there are 81 inmates? $\boxed{18}$

$y = 2/9\,(81) = 2 \cdot 9 = 18$

10d. If there are 12 officers on duty, how many inmates are there? $\boxed{54}$

$12 = \frac{2}{9}x$

$\frac{9}{2} \cdot 12 = 2/9\,x \cdot \frac{9}{2}$ → $54 = x$

11. As a child grows, an important measurement is the circumference of the child's head. The average circumference $C(t)$ of a male child's head is approximately a function of his age. This function can be represented by the following formula, where $C(t)$ is measured in centimeters (cm) and t is the child's age in months.

$$C(t) = \begin{cases} t + 35 & \text{, if } 0 \le t \le 12 \\ \frac{1}{6}t + 45 & \text{, if } 12 < t \le 36 \end{cases}$$

11a. What is the average circumference of a baby boy's head at birth? Express this information using function notation. $C(\underline{0}) = \underline{35}$

11b. What is the average circumference of a baby boy's head at one year? Express this information using function notation. $C(\underline{12}) = \underline{47}$ $12 + 35 = 47$

11c. Evaluate $C(24) =$ $\boxed{}$ Interpret this value in the context of the problem.

$C(24) = \frac{1}{6}(24) + 45 = 4 + 45 = 49$

The average circumference of a baby boy's head at 2 years is 49 cm.

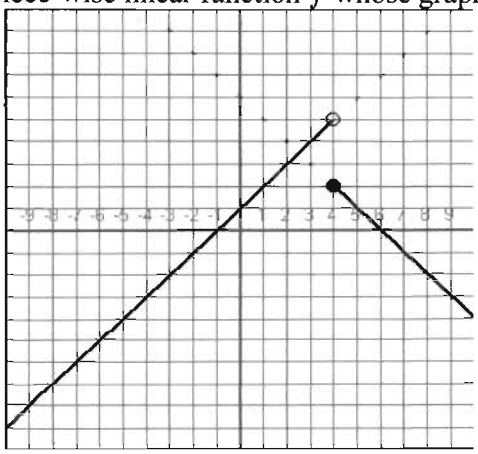
11d. What is the domain of C? Write your answer using interval notation. $\boxed{[0, 36]}$

12. Complete the formula for the piece-wise linear function f whose graph is shown here.

(0,1), (3,4)

$m = \frac{4-1}{3-0} = \frac{3}{3} = 1$

$y = x+1$



(4,2), (6,0)

$m = \frac{0-2}{6-4} = \frac{-2}{2} = -1$

$y-2 = -(x-4)$

$y = -x+4+2$

$y = -x+6$

$$f(x) = \begin{cases} \boxed{x+1} & \text{, if } x < \boxed{4} \\ \boxed{-x+6} & \text{, if } x \geq \boxed{4} \end{cases}$$

13. What is the y-intercept of the line whose formula is $x = 2$?

y-intercept = none (vertical line)

14. The percentage of doctorates in math in the U.S. earned by women was 7% in 1966. By 2006, the percentage had increased had increased to 30%.

14a. Write the formula for a linear function that models this data. Let y percentage of doctorates in math in the U.S. earned by women, and let x be the elapsed number of years from 1960.

$y =$ $\frac{23}{40}(x-6)+7$

(6,7), (46,30)

$m = \frac{30-7}{46-6} = \frac{23}{40}$

14b. Use the formula from part a to predict the percentage of doctorates in math in the U.S. earned by women in 2020. 38%

$y = \frac{23}{40}(60-6)+7 = \frac{23}{40}(54)+7 = 38.05$

15. What is the slope of a line that is parallel to the line $3x + 4y = 8$? Circle the correct answer.

i. 4 (ii.) $-\frac{3}{4}$ iii. $\frac{4}{3}$ iv. $\frac{8}{3}$ v. -3

$4y = -3x+8$

$y = \frac{-3}{4}x+2$

16. What is the formula of the line that is parallel to the line $3x + 4y = 8$ and passes through the point $(4, 2)$?

$y-2 = -\frac{3}{4}(x-4)$

$m = -3/4 \rightarrow 4/3$

17. Which of the following lines is perpendicular to the line $3x + 4y = 8$? Circle the correct answer.

i. $y = -\frac{1}{4}x - 2$ ii. $y = -\frac{3}{4}x - 1$ iii. $y = \frac{4}{3}x + 2$ iv. $y = \frac{3}{8}x + 3$ v. $y = \frac{1}{3}x + 1$

18. The amount of time y, in hours, needed to complete the yearly inventory at a certain company is inversely proportional to the number of persons x working on the inventory. Suppose it takes 5 hours to complete the inventory if 8 persons are working on it.

18a. Find the constant of proportionality. $k =$ 40

$y = \frac{k}{x}$

$5 = \frac{k}{8}$

$40 = k$

18b. Write the formula for the function.

$y =$ $40/x$

18c. How many hours will the inventory take if 20 persons are working on it? 2 hours

$y = \frac{40}{20} = 2$

19. A cell phone plan charges \$5.00 for the first 300 text messages a user sends each month and then \$0.15 for each additional text message over 300. Write the formula for a piece-wise linear function that calculates the cost y for sending x text messages in a month.

$$y = f(x) = \begin{cases} 5 & \text{, if } x \leq 300 \\ 0.15(x-300)+5 & \text{, if } x > 300 \end{cases}$$

20. A worker drops a screwdriver off a building under construction. The following function can be used to determine the height h of the screwdriver (in ft) after t sec.

$$h = 576 - 16t^2$$

20a. What is the y-intercept of this function? y-int. = 576 Interpret this value in the context of the problem.

The initial height of the screwdriver is 576 ft.

20b. What is the x-intercept of this function? x-int. = 6 Interpret this value in the context of the problem.

$0 = 576 - 16t^2$

$16t^2 = 576$

$t^2 = \frac{576}{16} = 36 \rightarrow t = \pm 6$

The screwdriver hits the ground after 6 sec.

20c. What is the domain of this function? Write your answer using interval notation.

$[0, 6]$

21. The daily profit P for Perry's Premium Chewing Tobacco is given by

$$P = 36x - 2x^2 - 22$$

where x represents the daily sales in hundreds of pounds, and the daily profit is measured in hundreds of dollars.

21a. Evaluate $P(0)$ and then interpret this value in the context of the problem.

$P(0) = -22$. Perry loses \$2200 on a day when no tobacco is sold.

21b. Fill in the blanks: The graph of P is a parabola that opens up.

Its coefficients are $a =$ -2, $b =$ 36, and $c =$ -22.

21c. What daily sales amount will lead to the maximum profit? What is the maximum daily profit?

sales amount = 900 lb

maximum profit = 14000 dollars

vertex $= \left(\frac{-36}{2(-2)} = \frac{-36}{-4} = 9,\ P(9)\right)$

$= 36 \cdot 9 - 2(9)^2 - 22$

$= 324 - 162 - 22 = 140)$

$= (9, 140)$

22. A parabola has its vertex at the point $(1, -7)$, its y-intercept is at -5, and its two x-intercepts are at approximately -0.9 and 2.9. Choose an appropriate graphing window for this parabola. Answers may vary.

$[-1, 3, 1]$ by $[-8, 2, 2]$

23. Consider the quadratic function with the formula $y = f(x) = 2x^2 - 4x - 5$. Show your work for the following questions.

$a=2,\ b=-4,\ c=-5$

23a. Find the y-intercept of f. y-int. = -5

23b. Find the coordinates of the vertex of f. vertex = $(1, -7)$

$\left(\frac{-(-4)}{2 \cdot 2} = \frac{4}{4} = 1,\ f(1) = -7\right)$

$= (1, -7)$

23c. Find the x-intercept(s) of f. First give exact answers, and then give answers rounded to one decimal place.

$\frac{2 \pm \sqrt{14}}{2}$; $-0.9, 2.9$

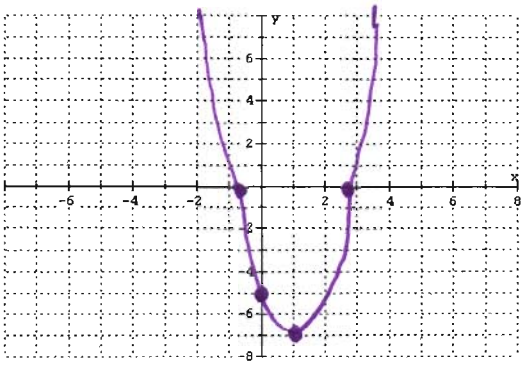
$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(-5)}}{2(2)} = \frac{4 \pm \sqrt{16+40}}{4} = \frac{4 \pm \sqrt{56}}{4} = \frac{4 \pm 2\sqrt{14}}{4} = \frac{2(2 \pm \sqrt{14})}{4} = \frac{2 \pm \sqrt{14}}{2}$

23d. Does the graph of the function f pass through the point $(5, 25)$? Yes ☑ No ☐

Explain. $25 = f(5) = 2(5)^2 - 4(5) - 5 = 50 - 20 - 5 = 25$, true

$\approx 2.9, -0.9$

23e. Use your answers from parts 23a - 23c to sketch a graph of the function f on the set of axes shown here.



24. Consider the quadratic function with the formula $y = g(x) = -x^2 + 3x - 1$. Show your work for the following questions.

$a=-1,\ b=3,\ c=-1$

24a. Find the y-intercept of f. y-int. = -1

24b. Find the coordinates of the vertex of g. vertex = $(3/2, 5/4)$

$\left(\frac{-3}{2(-1)} = \frac{-3}{-2} = \frac{3}{2},\right.$

$g(3/2) = -(3/2)^2 + 3(3/2) - 1$

$= -9/4 + 9/2 - 1 = 5/4\Big)$

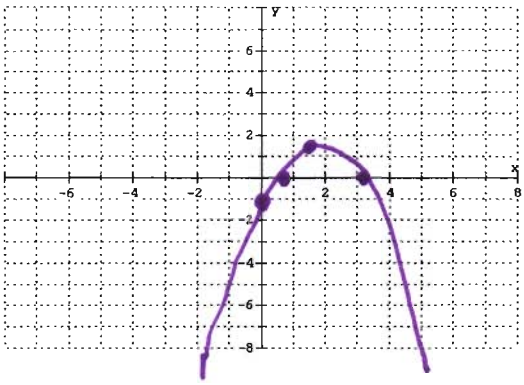
$= (3/2, 5/4)$

24c. Find the x-intercept(s) of g. First give exact answers, and then give answers rounded to one decimal place.

$3 \mp \sqrt{5}/2$ | $2.6, 0.4$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(-1)(-1)}}{2(-1)} = \frac{-3 \pm \sqrt{9-4}}{-2} = \frac{-3 \pm \sqrt{5}}{-2} = \frac{3 \mp \sqrt{5}}{2} \sim 2.6, 0.4$$

24d. Use your answers from parts 24a - 24c to sketch a graph of the function g on the set of axes shown here.



25. A parabola passes through the points $(-2, 1), (2, 2)$, and $(3, 1)$. Set up a 3 by 3 system of equations that can be used to find the coefficients a, b, and c of the parabola. *Do not solve.*

$$\begin{cases} 4a - 2b + c = 1 \\ 4a + 2b + c = 2 \\ 9a + 3b + c = 1 \end{cases}$$

$y = f(x) = ax^2 + bx + c$

$1 = f(-2) = a(-2)^2 + b(-2) + c$

$2 = f(2) = a(2)^2 + b(2) + c$

$1 = f(3) = a(3)^2 + b(3) + c$