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ABSTRACTS

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Plenary Talks (listed alphabetically by author)

Developments from selected conjectures of Graffiti
Speaker: Siemion Fajtowicz, University of Houston
Abstract: I will discuss selected conjectures of Graffiti, including some that led to fairly recent, many, or unusual developments. An example of the latter is a conjecture about stability of classical fullerenes that proved to be correct, in spite of an opinion of a fullerene expert. In the former category, there is a conjecture of my own (although directly inspired by Graffiti) linking Randic Index to the sum of positive eigenvalues and to the boiling point of alkanes. Other such examples are three new proofs of Euler Characteristic Formula inspired by an attempt of Stephanie Mathew - an UH undergraduate.
I will also discuss opinions and sometimes even false, or at very least, extremely controversial claims about what computers could or could not do, and even about what they have or have not done. To the extent to which it will be permitted by time, I will explain how Herbert Simon's claims that computer programs could not make original discoveries in physical sciences, led to the fullerene and other chemistry versions of Graffiti, and before that, to the idea of program accelerators, their limits, and how to cope with them.

Large even factors of a graph
Speaker: Odile Favaron, LRI, Université Paris Sud
Abstract: A factor of a graph G is a spanning subgraph of G and an even factor is a spanning subgraph, all vertices of which have positive even degree. In this talk we recall some generalities on factors and focus on even factors. It is known (Lovász, Fleischner) that every 2-edge-connected graph G with minimum degree at least 3 has an even factor. Moreover, Chen and Lai proved that under the same hypotheses G has an even factor with at least $\frac{2}{3}e_G$ edges, where e_G is the number of edges of G. We suppress these hypotheses and show that every simple graph G with an even factor has an even factor with at least $\frac{9}{16}e_G$ edges.

Searching for diamonds

Speaker: Jerrold R. Griggs, University of South Carolina

Abstract: We consider an extremal set theory analogue of Turán theory for graphs: Given a finite poset P, we consider the largest size $\mathrm{La}(n,P)$ of a family of subsets of $[n] := \{1,\ldots,n\}$ that contains no (weak) subposet P. Letting P_k denote the k-element chain (path poset), Sperner's Theorem (1928) gives that $\mathrm{La}(n,P_2) = \binom{n}{\lfloor n/2 \rfloor}$, and Erdős (1945) showed more generally that $\mathrm{La}(n,P_k)$ is the sum of the k middle binomial coefficients in n. Gyula Katona brought the attention of researchers to this problem for other posets P. It can be very challenging, even for small posets. With Lincoln Lu we conjecture that $\pi(P) := \lim_{n\to\infty} \mathrm{La}(n,P)/\binom{n}{\lfloor n/2 \rfloor}$ exists for general posets P, and, moreover, it is an integer. For $k \geq 2$ let D_k denote the k-diamond poset $\{A < B_1,\ldots,B_k < C\}$. By bounding the average number of times a random full chain meets a P-free family $\mathcal{F}$, with Wei-Tian Li and Lu we proved that $\pi(D_2) \leq 2.273$, if it exists. This is a stubborn open problem, since we expect $\pi(D_2) = 2$. It is then surprising that with Li we can explicitly determine $\pi(D_k)$ for infinitely many values of k, and, moreover, we can describe the extremal D_k-free families. In recent work with Andrew Dove we consider forbidding a family F of posets, which gives a pretty result when F consists of the three element V poset and its dual.

Recent results on the k-independence number of graphs

Speaker: Adriana Hansberg, UPC Barcelona

Abstract: Let $G = (V,E)$ be a graph and $k \geq 0$ an integer. A k-independent set $S \subseteq V$ is a set of vertices such that the maximum degree in the graph induced by S is at most k. With $\alpha_k(G)$ we denote the maximum cardinality of a k-independent set of G. In this talk, we will review some recent results on the k-independence number. Among them, we will present a result given in [Y. Caro, A. Hansberg, New approach to the k-independence number of a graph, Electron. J. Combin. 20(1) (2013), #P33.] which states that, for a graph G on n vertices and average degree d, $\alpha_k(G) \geq \frac{k+1}{\lceil d \rceil + k + 1} n$, improving the hitherto best general lower bound due to Caro and Tuza [Improved lower bounds on k-independence, J. Graph Theory 15 (1991), 99–107]. Moreover, the concepts of k-small set and k-large sets and their relation to the k-independence number will be introduced.

Some thoughts on switching reconstruction

Speaker: Brendan McKay, Australian National University

Abstract: The classical reconstruction problem asks when a graph G can be reconstructed from its deck, where the deck consists of cards showing each of the vertex-deleted subgraphs of G. Stanley, Bondy and others introduced variants of this problem, where the cards instead show the results of switching G. There are many possible meanings for "switching" and we consider several. For example, the edges of G might be coloured with two colours and a switching involves interchanging the colours at one vertex, or changing the colour of one edge. Or, G might be a digraph and switching reverses the directions at one vertex, or reverses one edge. In each case the cards show only equivalence classes of graphs, not labelled graphs, and the original graph needs to be reconstructed up to equivalence. Usually "equivalence" refers to graph isomorphism, but some interesting variants result from different definitions, such as considering the converse of a digraph to be equivalent to the original. We give some examples and partial solutions to switching reconstruction problems. We then present a uniform abstraction that captures all the proposed variations and produces some surprisingly strong general consequences. We also give some examples of how to find all the "switching-stable" graphs, which are those for which every switching produces a graph equivalent to the original. This is joint work with Beáta Faller, Paulette Lieby, Jeanette McLeod, and Pascal Schweitzer.

***k*-trees, *k*-frames, shells and independence polynomials**

Speaker: William Staton, University of Mississippi

Abstract: Much of what is known about trees extends smoothly to k-trees. Some fairly recent results of this type, concerning independence polynomials, will be presented. We discuss the 'shell' of a k-tree, corresponding to the line graph of a tree. A possibly new class of graphs, the k-frames will be defined, generalizing the k-trees.

Contributed Talks (listed alphabetically by author)

Coloring of the total graph of a commutative ring

Speaker: Ghodratollah Aalipour, University of Texas at El Paso

Abstract: Let R be a commutative ring with unity and $Z(R)$ be the set of zero-divisors of R. We denote by $T(\Gamma(R))$, the total graph of R, a simple graph with the vertex set R and two distinct vertices x and y are adjacent if and only if $x+y \in Z(R)$. In fact, $T(\Gamma(R))$ is the Cayley sum graph $Cay^{+}(R, Z(R))$. The induced subgraph of $T(\Gamma(R))$ on $Z(R)$ is denoted by $Z(\Gamma(R))$. These graphs were first introduced by D.F. Anderson and A. Badawi in 2008. In this talk, we apply some special combinatorial arrays to show that for a finite ring R, the chromatic number and clique number of $T(\Gamma(R))$ and $Z(\Gamma(R))$ are the same and indeed they are equal to $\max\{|m| : m \in \mathrm{Max}(R)\}$, provided that one of the following conditions holds:

(i) The residue field of R of minimum size has even characteristic.

(ii) Every residue field of R has odd characteristic and $\frac{R}{J(R)}$ has no summand isomorphic to $Z_3 \times Z_3$.

This is a joint work with Saieed Akbari.

On the total domination number and cut vertices of a graph

Speaker: David Amos, University of Houston-Downtown

Abstract: The total domination number γ_t of a graph is the cardinality of a smallest subset S of vertices such that every vertex of the graph is adjacent to a vertex in S. It is known that, for trees, $\gamma_t \geq (n - l + 2)/2$, where n is the order of the tree and l is the number of leaves. In this talk, we discuss how the above bound can be generalized to graphs. In particular, we show that for any graph with no isolated vertices, $\gamma_t \geq |C| - \mu(G[C]) + 1$, where $|C|$ is the number of cut vertices of the graph and $\mu(G[C])$ is the matching number of the subgraph induced by the set of cut vertices. We will also discuss some families of graphs that satisfy equality for this bound, as well as a characterization of the case of equality for trees. This is joint work with Ermelinda DeLaViña.

Complete r-partite graphs determined by their domination polynomial

Speaker: Barbara Anthony, Southwestern University

Abstract: The domination polynomial of a graph is the polynomial whose coefficients count the number of dominating sets of each cardinality. A recent question asks which graphs are uniquely determined (up to isomorphism) by their domination polynomial. We completely describe the complete r-partite graphs which are; in the bipartite case, this settles in the affirmative a conjecture of Aalipour, Akbari and Ebrahimi. This is joint work with Mike Picollelli.

Self-similar graphs and expanders

Speaker: Kiran Chilakamarri, Texas Southern University

Abstract: For any graph G on n vertices and for any symmetric subgraph J of $K_{n,n}$, we construct an infinite sequence of graphs $G^1, G^2, \cdots$ where $G^1 = G$ and having constructed G^{k-1}, G^k is obtained by replacing each vertex of G^{k-1} by a copy of G and each edge of G^{k-1} is replaced by the edge bundle J. We call these graphs Self-Similar graphs. We delineate those pairs (G, J) for which the chromatic numbers of $G^1, G^2, \cdots$ are bounded. When $G = K_n$ and J is a special matching we show that the sequence $G^1, G^2, \cdots$ is a sequence of expander graphs with vertex expansion coefficient of G^k is greater than $\frac{1}{2k}$. This is joint work with M.F. Khan, C.E. Larson, and C.J. Tymczak.

Consecutive interval embeddings and Wilf equivalence

Speaker: Garner Cochran, Trinity University

Abstract: Consider the alphabet A and define A^* as the set of words over A. Define a vector of sequences of subsets of $\mathbb{N}$ as $\vec{u} = (u_1, u_2, \ldots, u_k)$. Consider a word $w \in A^*$. Define their to be an embedding of $\vec{u}$ in w, $\vec{u} \leq w$ if there is some i such that, $w_i \in u_j, w_{i+1} \in u_{j+1}, \ldots w_{i+k-1} \in u_{j+k-1}$. Define a word that avoids the vector $\vec{u}$ as a word where there is no such i, such that $w_i \in u_j, w_{i+1} \in u_{j+1}, \ldots w_{i+k-1} \in u_{j+k-1}$.

We define the weight of a function as $\mathrm{wt}(w) = t^{|w|}x^{\sum(w)}$. We define the generating function for a certain pattern $\vec{u}$ as $F(u; x, t) = \sum_{u \leq w} \mathrm{wt}(w)$. We consider two patterns $\vec{u}$ and $\vec{v}$ to be Wilf Equivalent if $F(\vec{u}; x, t) = F(\vec{v}; x, t)$. We then prove some properties for Wilf Equivalence of patterns. We use these properties to then try to describe classes of Wilf Equivalent objects.

Properties on the k-residue of disjoint unions of graphs

Speaker: Randy Davila, Texas State University-San Marcos

Abstract: The k-residue of a graph, introduced by Jelen in a 1999 paper, is a lower bound on the k-independence number for every positive integer k. This generalized earlier work by Favaron, Maheo, and Sacle, by Griggs and Kleitman, and also by Triesch, who all showed that the independence number of a graph is at least as large as its Havel-Hakimi residue, defined by Fajtlowicz. While considering the k-residue of disjoint unions of graphs, we found that the k-residue of disjoint unions is at most the sum of k-residues of the connected components considered separately. In particular, we use this result to show an improvement on Jelens bound for connected graphs which have a maximum degree cut-vertex. Furthermore, we show that there are connected graphs for which this improves upon all known lower bounds on the k-independence number, including those of Hopkins and Staton and a recent result of Caro and Hansberg. This is joint work with Dr. Ryan Pepper, David Amos, and Randy Davila.

Weighted spanning tree enumerators of complete colorful complexes
Speaker: Art Duval, University of Texas at El Paso
Abstract: A complete colorful complex on r colors is an $(r-1)$-dimensional simplicial complex whose vertices are partitioned into r disjoint color classes, and whose facets are all the sets (of size r) containing exactly one vertex of each color. Adin enumerated the k-dimensional spanning trees of complete colorful complexes for all $k \leq r$, and Ehrenborg and van Willigenburg counted weighted spanning trees of complete bipartite graphs ($r = 2$). We find a factorization of a weighted enumeration of top-dimensional spanning trees of complete colorful complexes, for any r. The proof relies on the simplicial Matrix-Tree Theorem, and identification of factors, as in Martin and Reiner. This is a joint work with Ghodratollah Aalipour.

Dihedral Cayley directed strongly regular graphs
Speaker: Jonathan Gamez, University of Houston-Downtown
Abstract: A graph is a directed strongly regular graph (DSRG) if and only if the number of paths of length 2 from vertex x to vertex y is: λ, if there is an edge from x to y; μ, if there is not an edge from x to y (with x not equal to y); and t, if $x = y$. For every vertex in G, the in-degree and out-degree is k. The number of vertices in G is denoted by v. If G is a group and S a subset of G, then the Cayley graph is the directed graph whose vertices are the elements of G, and directed edges are (g, sg) for every g in G and for every s in S. If w is any natural number and $n = 4w + 2$, then we construct a family of DSRGs with parameters $v = 8w + 4,\ k = 4w,\ t = 2w,\ \mu = 2w,\ \textit{and}\ \lambda = 2w - 2$ utilizing Cayley graphs of the dihedral group D_{2n}. This is joint work with advisor Art Duval.

Inflection points of reliability polynomials
Speaker: Christy Graves, University of Texas at Tyler
Abstract: In this talk we give a class of graphs whose reliability polynomials have arbitrarily many inflection points. This is joint work with David Milan.

Counting fullerene patches
Speaker: Stephen Graves, University of Texas at Tyler
Abstract: A method for canonically embedding a fullerene patch in the hexagonal tessellation of the plane is described, allowing direct computation for the number of nonisomorphic fullerene patches with a given boundary code.

Degree sequences and k-edge-connected uniform hypergraphs
Speaker: Xiaofeng Gu, Texas State University
Abstract: An integral sequence d is r-uniform hypergraphic if there is a simple r-uniform hypergraph H with degree sequence d, and such a hypergraph H is a realization of d. In this paper, it is proved that an r-uniform hypergraphic sequence $d = (d_1, d_2, \cdots, d_n)$ has a k-edge-connected realization if and only if both $d_i \geq k$ for $i = 1, 2, \cdots, n$ and $\sum_{i=1}^{n} d_i \geq \frac{r(n-1)}{r-1}$, which generalizes the formal result of Edmonds for graphs and that of Boonyasombat for hypergraphs. This is a joint work with Dr. Hong-Jian Lai at West Virginia University.

A comparison between the zero forcing number and the strong metric dimension of graphs
Speaker: Cong Kang, Texas A&M University at Galveston
Abstract: The *zero forcing number*, $Z(G)$, of a graph G has recently become an interesting graph parameter studied in its own right since its introduction by the "AIM Minimum Rank-Special Graphs Work Group". The *strong metric dimension*, $sdim(G)$, of a graph G is the minimum among cardinalities of all strong resolving sets: $S \subseteq V(G)$ is a *strong resolving set* if for any $u, v \in V(G)$, there exists an $x \in S$ such that either u lies on an $x - v$ geodesic or v lies on an $x - u$ geodesic. In this paper, we prove that $Z(G) \leq sdim(G)$ when G is a tree or a unicyclic graph, and we characterize trees T attaining $Z(T) = sdim(T)$. It is easy to see that $sdim(T + e) - sdim(T)$ can be arbitrarily large for a tree T; we prove that $sdim(T + e) \geq sdim(T) - 2$ and show that the bound is sharp. We conclude with the open problem of seeking a refinement to $Z(G) \leq sdim(G) + 3r(G)$, an inequality proven in the final section and where $r(G)$ is the cycle rank of G. This is a joint work with Eunjeong Yi.

On the 2-independence number and the neighborhood of the core of a graph
Speaker: Iride Lazo, University of Houston-Downtown
Abstract: The k-independence number of a graph is the cardinality of a largest set of vertices that induce a subgraph of maximum degree at most $k - 1$. The core of the graph is the intersection of all maximum 1-independent sets, in other words, the intersection of all maximum independent sets of the graph. We prove two conjectures made by the computer program Graffiti.pc relating the 2-independence number of a graph to invariants of the subgraph induced by the neighborhood of the core of the graph. This is joint work with Ermelinda DeLaViña.

Enumeration of finite inverse semigroups
Speaker: Martin Malandro, Sam Houston State University
Abstract: Just as groups are algebraic structures which encode global symmetries, inverse semigroups are algebraic structures which encode partial symmetries. However, unlike groups, the number $S(n)$ of inverse semigroups of order n (up to isomorphism) increases rapidly as n increases. The set $E(S)$ of idempotents of an inverse semigroup S forms a meet-semilattice. We present a fast algorithm that takes as input a finite meet-semilattice E and an integer n and enumerates up to isomorphism the inverse semigroups S of order n such that $E(S) = E$. We then iterate over the meet-semilattices of order $1, \dots, n$ to compute $S(n)$ for some small values of n.

Some notes on prime labelings
Speaker: Alison Marr, Southwestern University
Abstract: This talk will discuss prime labelings of graphs. A prime labeling of a graph on v vertices is an assignment of the integers $\{1, 2, \dots, v\}$ to the vertices of graph such that if there is an edge between any two vertices, the labels on those vertices must be relatively prime. This talk will discuss some new labelings of ladders and complete bipartite graphs as well as present some open problems. This is joint work with Adam Berliner, Nate Dean, Jonelle Hook, Aba Mbirika, and Cayla McBee.

Crossings and nestings in bit-strings
Speaker: Mitch Phillipson, Texas A&M University
Abstract: Crossings and nestings have been studied extensively in structures such as matchings, partitions and permutations. In this talk we discuss some of the results of these structures and detail new work on bit-strings.

The combinatorics of complex hermite polynomials
Speaker: Plamen Simeonov, University of Houston-Downtown
Abstract: We will present several known and and several new combinatorial interpretations of complex (bi-variate) and multivariate Hermite polynomials. We use these interpretations to derive known and new orthogonality relations for these Hermite polynomials. This is part of a joint work with Mourad Ismail from the University of Central Florida.

On the maximum order of k-independent sets
Speaker: Bill Waller, University of Houston-Downtown
Abstract: A well-known folk theorem, which can also be considered a corollary of a theorem of Ore, states that the maximum order of an independent set of vertices in a simple, connected graph is no more than the order of the complement of a minimum dominating set of vertices. In this paper we prove a generalization of this folk theorem which sometimes gives an improved upper bound on the independence number. The result also extends to the maximum order of k-independent sets. This is a joint work with Ryan Pepper.

Decision trees, monotone functions, and monotone graph invariants
Speaker: Jacob White, Texas A&M University
Abstract: The monotone evasiveness conjecture states that any algorithm which determines a graph property by using a decision tree must run in time $\binom{n}{2}$. That is, in the worst case the entire structure of a graph must be known before a yes/no answer can be reached.

Chains of length 2 in fillings of layer polyominoes
Speaker: Catherine Yan, Texas A&M University
Abstract: The symmetry of northeast and southeast chains of length 2 in 01-fillings of moon polyominoes was characterized by Kasraoui who unified recent results on the symmetry of the numbers of crossings and nestings in various combinatorial structures, including permutations, words, matchings, set partitions, linked partitions, and certain families of graphs. In this talk we extend this symmetry to fillings of a more general family of polyominoes – layer polyominoes, which are intersection-free and row-convex, but not necessarily column-convex.

Nordhaus-Gaddum type results on metric dimension and strong metric dimension of graphs
Speaker: Eunjeong Yi, Texas A&M University at Galveston
Abstract: The *metric dimension*, $\dim(G)$, of a graph G is the minimum number of vertices such that all vertices are uniquely determined by their distances to the chosen vertices. The *strong metric dimension*, $sdim(G)$, of a graph G is the minimum among cardinalities of all strong resolving sets: $S \subseteq V(G)$ is a *strong resolving set* if for any $u, v \in V(G)$, there exists an $x \in S$ such that either u lies on an $x - v$ geodesic or v lies on an $x - u$ geodesic. We discuss Nordhaus-Gaddum type results on metric dimension and strong metric dimension of graphs. Nordhaus-Gaddum type result on metric dimension is a joint work with Linda Eroh and Cong X. Kang.