Sigma Notation

Definition : If a_m, a_{m+1}, $\ldots$, a_n are real numbers and m and n are integers such that $m \leq n$, then

$$\sum_{i=m}^{n} a_i = a_m + a_{m+1} + a_{m+2} + \ldots + a_{n-1} + a_n$$

Theorem : Let c be a constant and n be a positive integer. Then

(1) $\displaystyle\sum_{i=m}^{n} ca_i = c\sum_{i=m}^{n} a_i$

(2) $\displaystyle\sum_{i=m}^{n} (a_i + b_i) = \sum_{i=m}^{n} a_i + \sum_{i=m}^{n} b_i$

(3) $\displaystyle\sum_{i=m}^{n} (a_i - b_i) = \sum_{i=m}^{n} a_i - \sum_{i=m}^{n} b_i$

(4) $\displaystyle\sum_{i=1}^{n} 1 = n$

(5) $\displaystyle\sum_{i=1}^{n} c = nc$

(6) $\displaystyle\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

(7) $\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

(8) $\displaystyle\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$

(9) $\displaystyle\sum_{i=1}^{n} i^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$