Group Work 2, Section 4.9
The Root-Finding Race

As you have just seen with the function $f(x) = \cos x - x$, Newton's method is frequently a very efficient way of finding a root to an equation. But Newton's method doesn't always work. Sometimes it gives you an answer you don't want, sometimes it can't be used, and sometimes it takes you further and further away from the root you are looking for!

Consider $f(x) = 3 + 2x - x^2$.

Use algebra to find the positive roots of f.

Now let's say that someone wanted to try Newton's method out on the above problem. What would happen if they started with $x_0 = 0$? What would happen if they started with $x_0 = 1$?

Suppose $f(x)$ was a very complicated function with a lot of roots in a very small interval. Using the example above, what problems could arise in trying to use Newton's method to find all the roots?

Now we consider an even worse scenario: one where Newton's method takes you further away from the only root of a function, rather than closer. In order to understand the circumstances which cause this to happen, consider the following: Suppose you are at a point x where the function is positive but the derivative is negative. Will the next iteration give you a number larger or smaller than x?

Let $f(x) = \dfrac{x}{1+x^2}$. What is the only root of f?

Graph f on the interval $[0, 10]$ using technology. Notice that it is first increasing, and then decreasing. What is the global maximum of f?

What happens if we use Newton's method starting at $x_0 = 2$? Can you explain why Newton's method "thinks" it is approaching a root?