Group Work 1, Section 4.9
The Problem of Archimedes

The brilliant Greek engineer Archimedes (287 B.C.–212 B.C.) was concerned with using a plane to cut a sphere into two pieces, one of which had a volume of twice the other. He found a formula for the volume of a section of a given height:

$$V = \tfrac{1}{3}\pi h^2 (3r - h)$$

where h is the height of the section and r is the radius of the sphere.

1. If a plane at distance $x = r - h$ from the center of the unit sphere cuts it into two segments, one with twice the volume of the other, show that x must satisfy the equation

$$3x^3 - 9x + 2 = 0$$

2. Now use Newton's method to find a solution x to the problem of Archimedes, accurate to 8 decimal places.