Applications of Integration

Areas between curves

The area of the region bounded by the curves $y = f(x)$, $y = g(x)$, and the lines $x = a$ and $x = b$, where f and g are continuous and $f(x) \geq g(x)$ for all $x \in [a, b]$ is :

$$A = \int_a^b [f(x) - g(x)]\, dx$$

Volume

The volume of the solid obtained by revolving the region bounded by $y = f(x)$, $y = 0$, $x = a$, and $x = b$ about the x-axis is :

$$V = \int_a^b \pi [f(x)]^2\, dx$$

The volume of the solid obtained by revolving the region bounded by $x = g(y)$, $x = 0$, $y = c$, and $y = d$ about the y-axis is :

$$V = \int_c^d \pi [g(y)]^2\, dy$$

(The **washer method**) The volume of the solid generated by revolving the region bounded by $y = f(x)$, $y = g(x)$, $x = a$, and $x = b$ [where $f(x) \geq g(x)$] about the x-axis is :

$$V = \pi \int_a^b \{[f(x)]^2 - [g(x)]^2\}\, dx$$

(The **method of cylindrical shells**) The volume of the solid generated by revolving the region bounded by $y = f(x)$, $y = 0$, $x = a$, and $x = b$ about the y-axis is :

$$V = \int_a^b 2\pi x f(x)\, dx \text{ where } 0 \leq a < b$$