NAME_____

MATH 2401 PRACTICE QUIZ 3

1. Find the derivative of the function using the definition of derivative.

$$f(x) = \sqrt{3x+1}$$

2. Differentiate each function.

(1) $f(x) = \dfrac{3}{x^5} - \dfrac{7}{4}x^4 + x + \sqrt{2}$

(2) $f(x) = 3x^2(5x-1)^{10}$

(3) $f(x) = \dfrac{4x-5}{3x^2+2}$

(4) $f(x) = \sqrt[3]{x^3+3x}$

(5) $f(x) = sec\, x + tan^2 x$

3. Find each limit.

(1) $\lim_{\theta \to 0} \frac{sin\, 7\theta}{\theta}$

(2) $\lim_{\theta \to \pi/2} \frac{cos\, \theta - 1}{\theta}$

4. Consider the function $f(x) = x sin\, x + cos\, x$:

(1) Find $f'(x)$.

(2) Find the equation of the tangent line to the curve at $(\pi,\ f(\pi))$.

Click here for the answers

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1. Find the derivative of the function using the definition of derivative.

$f(x) = \sqrt{3x+1}$

$$\lim_{h\to 0}\frac{\sqrt{3(x+h)+1}-\sqrt{3x+1}}{h} = \lim_{h\to 0}\frac{\sqrt{3x+3h+1}-\sqrt{3x+1}}{h} =$$

$$\lim_{h\to 0}\frac{\sqrt{3x+3h+1}-\sqrt{3x+1}}{h}\cdot\frac{\sqrt{3x+3h+1}+\sqrt{3x+1}}{\sqrt{3x+3h+1}+\sqrt{3x+1}} =$$

$$\lim_{h\to 0}\frac{3x+3h+1-(3x+1)}{h(\sqrt{3x+3h+1}+\sqrt{3x+1})} = \lim_{h\to 0}\frac{3x+3h+1-3x-1}{h(\sqrt{3x+3h+1}+\sqrt{3x+1})} =$$

$$\lim_{h\to 0}\frac{3h}{h(\sqrt{3x+3h+1}+\sqrt{3x+1})} = \lim_{h\to 0}\frac{3}{\sqrt{3x+3h+1}+\sqrt{3x+1}} =$$

$$\frac{3}{\sqrt{3x+3\cdot 0+1}+\sqrt{3x+1}} = \frac{3}{\sqrt{3x+1}+\sqrt{3x+1}} = \frac{3}{2\sqrt{3x+1}}$$

2. Differentiate each function.

(1) $f(x) = \dfrac{3}{x^5} - \dfrac{7}{4}x^4 + x + \sqrt{2} = 3x^{-5} - \dfrac{7}{4}x^4 + x + \sqrt{2}$

$$f'(x) = 3(-5)x^{-6} - \frac{7}{4}\cdot 4x^3 + 1 + 0 = \frac{-15}{x^6} - 7x^3 + 1$$

(2) $f(x) = 3x^2(5x-1)^{10}$

$$f'(x) = 3\cdot 2x(5x-1)^{10} + 3x^2\cdot 10(5x-1)^9\cdot 5 =$$

$$6x(5x-1)^{10} + 150x^2(5x-1)^9 = 6x(5x-1)^9(5x-1+25x) =$$

$$6x(5x-1)^9(30x-1)$$

(3) $f(x) = \dfrac{4x-5}{3x^2+2}$

$$f'(x) = \frac{4(3x^2+2)-(4x-5)\cdot 6x}{(3x^2+2)^2} = \frac{12x^2+8-24x^2+30x}{(3x^2+2)^2} =$$

$$\frac{-12x^2+30x+8}{(3x^2+2)^2}$$

(4) $f(x) = \sqrt[3]{x^3+3x} = (x^3+3x)^{1/3}$

$$f'(x) = \frac{1}{3}(x^3+3x)^{1/3-1}(3x^2+3) = \frac{1}{3}(x^3+3x)^{-2/3}\cdot 3(x^2+1) =$$

$$\frac{x^2+1}{(x^3+3x)^{2/3}} = \frac{x^2+1}{\left(\sqrt[3]{x^3+3x}\right)^2}$$

(5) $f(x) = sec\,x + tan^2 x$

$$f'(x) = sec\,x\,tan\,x + 2tan\,x\,sec^2 x =$$

$$sec\,x\,tan\,x(1+2sec\,x)$$

3. Find each limit.

(1) $\lim_{\theta \to 0} \frac{sin\, 7\theta}{\theta} = \lim_{\theta \to 0} \frac{7 sin\, 7\theta}{7\theta} = 7 \lim_{\theta \to 0} \frac{sin\, 7\theta}{7\theta} = 7 \cdot 1 = 7$

(2) $\lim_{\theta \to \pi/2} \frac{cos\, \theta - 1}{\theta} = \frac{cos\, (\pi/2) - 1}{\pi/2} = \frac{0 - 1}{\pi/2} = -\frac{2}{\pi}$

4. Consider the function $f(x) = x sin\, x + cos\, x$:

(1) Find $f'(x)$.

$f'(x) = 1 \cdot sin\, x + x cos\, x - sin\, x = x cos\, x$

(2) Find the equation of the tangent line to the curve at $(\pi,\ f(\pi))$.

$The\ slope\ of\ the\ tangent\ line\ to\ the\ curve\ at\ x = \pi\ is$

$f'(\pi) = \pi \cdot cos\, \pi = \pi(-1) = -\pi$

$f(\pi) = \pi \cdot sin\, \pi + cos\, \pi = \pi \cdot 0 + (-1) = -1$

$So,\ the\ equation\ of\ the\ tangent\ line\ to\ the\ curve\ at$

$(\pi,\ f(\pi))\ is \quad y - (-1) = -\pi(x - \pi)$

$y + 1 = -\pi x + \pi^2$

$y = -\pi x + \pi^2 - 1$