College Algebra
Workshop 5 - Part 3

Unit 5 (Part 3) Piecewise-defined Functions and Exponential Functions

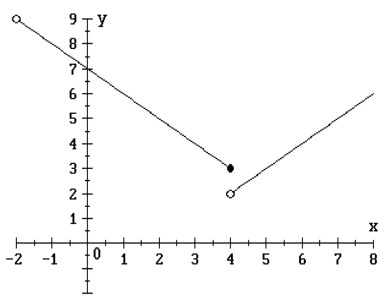
1. Let $F(x) = \begin{cases} x^2, & \text{if } 0 < x < 2 \\ 4, & \text{if } 2 \le x < 4 \\ 7 - x, & \text{if } 4 \le x \le 6 \end{cases}$

a. Evaluate each of the following outputs *by hand*:
i. $F(2)$ **ii.** $F(4)$ **iii.** $F(3)$ **iv.** $F(F(3))$

b. Write the domain of F.

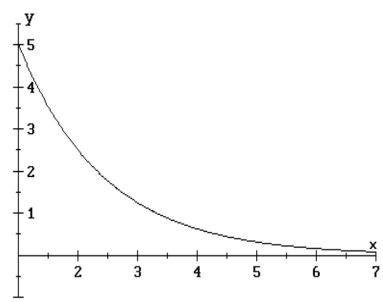
For each function graphed in Problems 2 and 3, state the graphing window used and then explain why the graph is *not* a good graph of the function.

2.



Graph of $y = \begin{cases} 10 & , \text{ if } x \le -2 \\ |x - 7| & , \text{ if } -2 < x \le 4 \\ x - 2 & , \text{ if } x > 4 \end{cases}$

3.



Graph of $y = 10\left(\frac{1}{2}\right)^x$

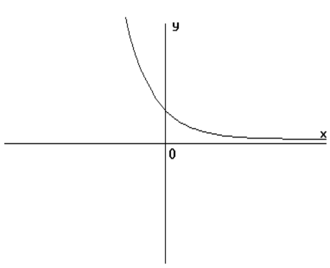
4. Suppose the exponential function B has the symbol rule $B(x) = 4\left(\frac{3}{2}\right)^x$.

a. Evaluate each of the following outputs *by hand*:
i. $B(-2)$ **ii.** $B(0)$ **iii.** $B(2)$ **iv.** $B(4)$

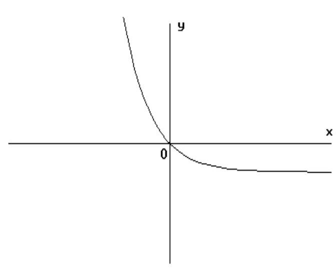
b. If a table is made for B where the input numbers increase by a constant amount, then the output values will ___?___ .
i. Increase by a constant amount **ii.** Increase by a constant percentage
iii. Decrease by a constant amount **iv.** Decrease by a constant percentage

c. Which of the following graphs *could be* the graph of the function B?

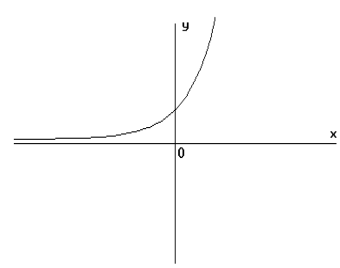
i.



ii.



iii.



iv. None of these

5. For each of the following quantities that is growing or decaying by a constant percentage, find the growth factor for the quantity.
a. The quantity is growing by 8% per year.

b. The quantity is growing by $\frac{1}{4}\%$ per month.

c. The quantity is decaying by 12% per year.

d. For each day that passes, the amount of the quantity remaining is 30% of the amount on the previous day.

6. A sample of the radioactive element radium loses 4% of its mass each *century*. Let's assume that the function $A(t)$ outputs the amount remaining of an initial mass of 10 grams of radium after t centuries.
a. Why is it appropriate to assume that A is an exponential function?

b. Find the values of the initial value c and the growth factor b for the function A. Then write the symbol rule for A.

c. Find the amount of radium remaining after 3 centuries.

d. Use a computer or calculator to sketch a good graph of A.

e. Use the graph from part d to find the half-life of radium. That is, after what amount of time will the remaining amount of the initial mass of radium be 5 grams?

f. In part e, you have in effect solved an equation. Write this equation.

In Practice Exercises 7–10, *graphically* estimate the solutions to each of the given equations and inequalities.
7. $1.2e^x = 5.5$

8. $1.2e^x \geq 5.5$

9. $\left(\frac{3}{4}\right)^x = 10$

10. $\left(\frac{3}{4}\right)^x - 10 < 0$