WOW 211 and 212

WOW 211 and 212. There is a simple connected graph G with 25 vertices, the set A of vertices of G of minimal degree has $|N(A)| = |N(A) - A| = 16$, the lower median of the degree sequence of the complementary graph $\overline{G}$ is 8, but G does not have a Hamiltonian path.

Solution by Richard Stong, CCR, La Jolla. Let G be the graph with vertices $V(G) = \{v_{0,0}, v_{0,1}, \ldots, v_{3,3}, w_0, w_1, w_2, x_1, \ldots, x_6\}$ such that

(i) The $v_{i,j}$ form a complete graph.
(ii) Each w_i is adjacent to only $v_{i,j}$, $0 \le j \le 3$.
(iii) Each x_k is adjacent to only $v_{3,j}$, $0 \le j \le 3$.

Clearly G is connected and simple. The vertices $v_{i,j}$ have degree 16 if $i < 3$ and 21 if $i = 3$. The vertices w_i and x_k all have degree 4. Therefore $A = \{w_0, w_1, w_2, x_1, \ldots, x_m\}$ is the set of vertices of minimal degree and we have $N(A) = \{v_{0,0}, v_{0,1}, \ldots, v_{3,3}\}$. Hence $|N(A)| = |N(A) - A| = 16$, as claimed. The degree sequence of G is $4^9 16^{12} 21^4$ hence the degree sequence of $\overline{G}$ is $3^4 8^{12} 20^9$ and the median of this degree sequence is 8. To see that G does not have a Hamiltonian path simply note that removing the 4 vertices $v_{3,j}$ leaves 7 components, the six isolated vertices x_k and one large component.