TOTAL DOMINATION OF AT MOST CUBIC CONNECTED GRAPHS

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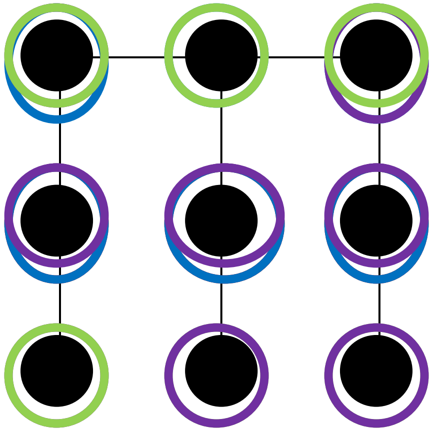
DEFINITIONS



Domination number (γ)

Total Domination number (γ_t)

Two Domination number (γ_2)

Three Domination number (γ_3)

TOTAL DOMINATION

- Cockayne, E.J., Dawes, R.M., Hedetniemi, S.T., *Total domination in graphs*. Networks, 10:211-219, 1980.

GRAFFITI.PC

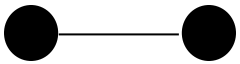
- Computer program written by Dr. Ermelinda DeLaViña
  - Creates conjectures for specific graph parameters
    - Total domination for at most cubic connected graphs

PREVIOUS WORK SUMMER 2010

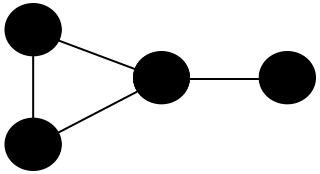
- Graph theory directed study
  - Introduced to terms, notation, and proof techniques

- First conjectures from Graffiti.pc

$$\gamma_t = \Delta(G)$$

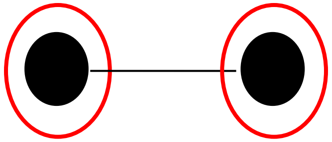


$\gamma_t \leq$ The number of minimum degree vertices



SIMPLE COUNTEREXAMPLES

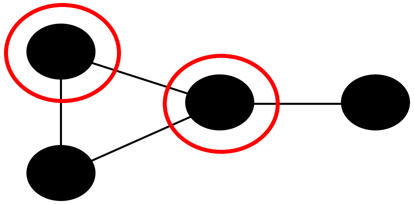
$$\gamma_t = \Delta(G)$$



$$\gamma_t = 2$$

$$\Delta(G) = 1$$

$$\gamma_t \leq \# of \delta(v)$$



$$\gamma_t = 2$$

$$\# of \delta(v) = 1$$

KNOWN FACTS

- $\gamma_t \leq \frac{2}{3}n$
  - Cockayne, E.J., Dawes, R.M., & Hedetniemi, S.T., *Total domination in graphs*. Networks 10:211-219, 1980.
- Total domination not bounded by 2 domination
  - Paths and cycles are some exceptions
- Interestingly Graffiti.pc conjectured the following:

MAJOR FOCUS

- **Conjecture.** Let *G* be a connected at most cubic graph. Then

$$\gamma_t(G) \leq 2 * \left\lfloor \frac{\gamma_3(G)}{2} \right\rfloor$$

- Simplified as $\gamma_t \leq \gamma_3$

- Several partial results have been found

PARTIAL RESULTS

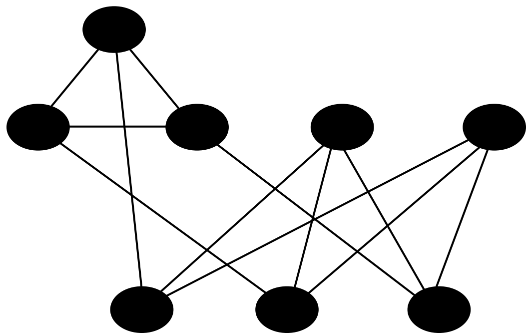
- *If* $\frac{2}{3}n \leq \gamma_3$, *then* $\gamma_t(G) \leq \gamma_3(G)$

- *If G is at most cubic then,* $\frac{n}{2} \leq \gamma_3$

MY CONJECTURE

Conjecture (Jenkins) Let *G* be an at most cubic connected *n*-vertex graph.

Then *G* is 3-regular if and only if $\frac{n}{2} = \gamma_3(G)$



The implication

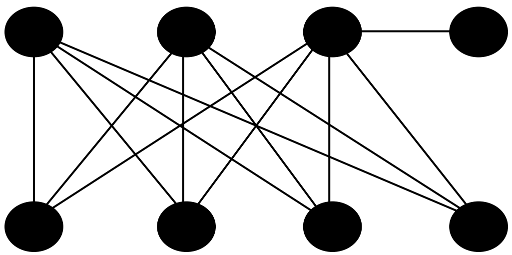
if *G* is 3-regular, then $\frac{n}{2} = \gamma_3(G)$

Is false as seen in the graph above.

ON THE OTHER HAND

- ***Proposition**. Let G be an at most cubic connected n-vertex graph.*
  If $\gamma_3(G) = \frac{n}{2}$ *, then G is 3-regular*

  Follow-up question

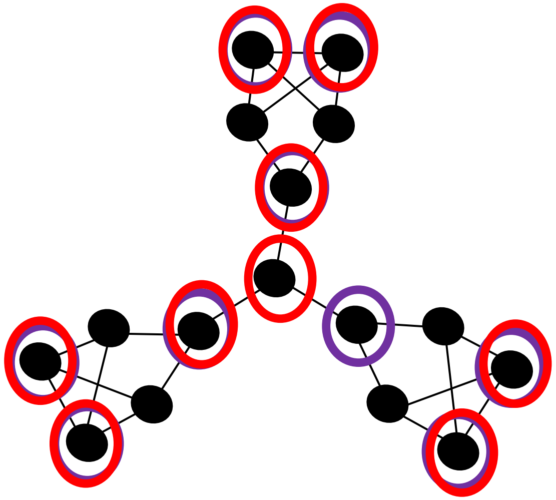


For a graph that is not restricted to at most cubic, the above graph is simple counterexample

ANOTHER PARTIAL RESULT

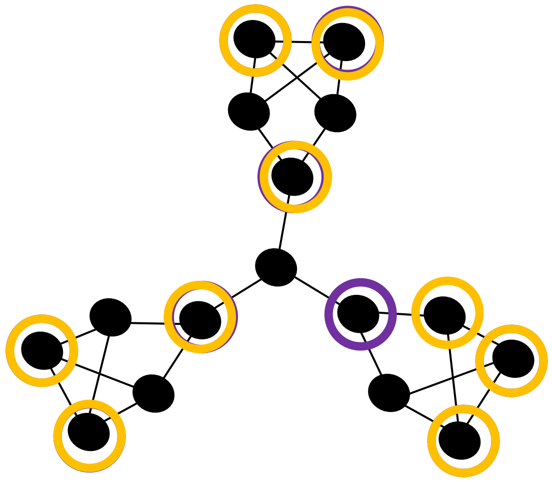
- ***Proposition***. Let D_3 be a smallest 3-dominating set for *G*. If there is only one $v \in D_3$ not dominated by a vertex in D_3 then $\gamma_t \leq \gamma_3$

IDEA OF 'SWAPPING'



Total dominating set
3-dominating set

ISSUES WITH 'SWAPPING'



Error set

3-dominating set

Closing Comments

- Disproved 25 different conjectures from Graffiti.pc
- Proved several partial results for $\gamma_t \leq \gamma_3$
- Found a partial result in a 2010 paper of Chellali et. Al, If *G* is an at most cubic tree different than a star, then $\gamma_3 \geq \gamma_t + 2$
- Future work could improve ‘swapping’ technique

THANK YOU