1.9 Exercises

1–6 ■ Use a graphing calculator or computer to decide which viewing rectangle (a)–(d) produces the most appropriate graph of the equation.

1. $y = x^4 + 2$
   (a) $[-2, 2]$ by $[-2, 2]$
   (b) $[0, 4]$ by $[0, 4]$
   (c) $[-8, 8]$ by $[-4, 40]$
   (d) $[-40, 40]$ by $[-80, 800]$

2. $y = x^2 + 7x + 6$
   (a) $[-5, 5]$ by $[-5, 5]$
   (b) $[0, 10]$ by $[-20, 100]$
   (c) $[-15, 8]$ by $[-20, 100]$
   (d) $[-10, 3]$ by $[-100, 20]$

3. $y = 100 - x^2$
   (a) $[-4, 4]$ by $[-4, 4]$
   (b) $[-10, 10]$ by $[-10, 10]$
   (c) $[-15, 15]$ by $[-30, 110]$
   (d) $[-4, 4]$ by $[-30, 110]$

4. $y = 2x^2 - 1000$
   (a) $[-10, 10]$ by $[-10, 10]$
   (b) $[-10, 10]$ by $[-100, 100]$
   (c) $[-10, 10]$ by $[-1000, 1000]$
   (d) $[-25, 25]$ by $[-1200, 200]$

5. $y = 10 + 25x - x^3$
   (a) $[-4, 4]$ by $[-4, 4]$
   (b) $[-10, 10]$ by $[-10, 10]$
   (c) $[-20, 20]$ by $[-100, 100]$
   (d) $[-100, 100]$ by $[-200, 200]$

6. $y = \sqrt{8x - x^2}$
   (a) $[-4, 4]$ by $[-4, 4]$
   (b) $[-5, 5]$ by $[0, 100]$
   (c) $[-10, 10]$ by $[-10, 40]$
   (d) $[-2, 10]$ by $[-2, 6]$

7–18 ■ Determine an appropriate viewing rectangle for the equation and use it to draw the graph.

7. $y = 100x^2$
8. $y = -100x^2$
9. $y = 4 + 6x - x^2$
10. $y = 0.3x^2 + 1.7x - 3$
11. $y = \sqrt[4]{256 - x^2}$
12. $y = \sqrt{12x - 17}$
13. $y = 0.01x^3 - x^2 + 5$
14. $y = x(x + 6)(x - 9)$
15. $y = x^4 - 4x^3$
16. $y = \dfrac{x}{x^2 + 25}$
17. $y = 1 + |x - 1|$
18. $y = 2x - |x^2 - 5|$

19. Graph the circle $x^2 + y^2 = 9$ by solving for y and graphing two equations as in Example 3.

20. Graph the circle $(y - 1)^2 + x^2 = 1$ by solving for y and graphing two equations as in Example 3.

21. Graph the equation $4x^2 + 2y^2 = 1$ by solving for y and graphing two equations corresponding to the negative and positive square roots. (This graph is called an *ellipse*.)

22. Graph the equation $y^2 - 9x^2 = 1$ by solving for y and graphing the two equations corresponding to the positive and negative square roots. (This graph is called a *hyperbola*.)

23–26 ■ Do the graphs intersect in the given viewing rectangle? If they do, how many points of intersection are there?

23. $y = -3x^2 + 6x - \frac{1}{2}$, $y = \sqrt{7 - \frac{7}{12}x^2}$; $[-4, 4]$ by $[-1, 3]$

24. $y = \sqrt{49 - x^2}$, $y = \frac{1}{5}(41 - 3x)$; $[-8, 8]$ by $[-1, 8]$

25. $y = 6 - 4x - x^2$, $y = 3x + 18$; $[-6, 2]$ by $[-5, 20]$

26. $y = x^3 - 4x$, $y = x + 5$; $[-4, 4]$ by $[-15, 15]$

27–36 ■ Solve the equation both algebraically and graphically.

27. $x - 4 = 5x + 12$
28. $\frac{1}{2}x - 3 = 6 + 2x$
29. $\dfrac{2}{x} + \dfrac{1}{2x} = 7$
30. $\dfrac{4}{x + 2} - \dfrac{6}{2x} = \dfrac{5}{2x + 4}$
31. $x^2 - 32 = 0$
32. $x^3 + 16 = 0$
33. $16x^4 = 625$
34. $2x^5 - 243 = 0$
35. $(x - 5)^4 - 80 = 0$
36. $6(x + 2)^5 = 64$

37–44 ■ Solve the equation graphically in the given interval. State each answer correct to two decimals.

37. $x^2 - 7x + 12 = 0$; $[0, 6]$
38. $x^2 - 0.75x + 0.125 = 0$; $[-2, 2]$
39. $x^3 - 6x^2 + 11x - 6 = 0$; $[-1, 4]$
40. $16x^3 + 16x^2 = x + 1$; $[-2, 2]$
41. $x - \sqrt{x + 1} = 0$; $[-1, 5]$
42. $1 + \sqrt{x} = \sqrt{1 + x^2}$; $[-1, 5]$
43. $x^{1/3} - x = 0$; $[-3, 3]$
44. $x^{1/2} + x^{1/3} - x = 0$; $[-1, 5]$

45–48 ■ Find all real solutions of the equation, correct to two decimals.

45. $x^3 - 2x^2 - x - 1 = 0$
46. $x^4 - 8x^2 + 2 = 0$
47. $x(x - 1)(x + 2) = \frac{1}{6}x$
48. $x^4 = 16 - x^3$

49–56 ■ Find the solutions of the inequality by drawing appropriate graphs. State each answer correct to two decimals.

*49. $x^2 - 3x - 10 \le 0$

50. $0.5x^2 + 0.875x \le 0.25$

51. $x^3 + 11x \le 6x^2 + 6$

52. $16x^3 + 24x^2 > -9x - 1$

*53. $x^{1/3} < x$

54. $\sqrt{0.5x^2 + 1} \le 2|x|$

55. $(x + 1)^2 < (x - 1)^2$

56. $(x + 1)^2 \le x^3$

57. In Example 6 we found two solutions of the equation $x^3 - 6x^2 + 9x = \sqrt{x}$, the solutions that lie between 1 and 6. Find two more solutions, correct to two decimals.

Applications

58. Estimating Profit An appliance manufacturer estimates that the profit y (in dollars) generated by producing x cooktops per month is given by the equation

$$y = 10x + 0.5x^2 - 0.001x^3 - 5000$$

where $0 \le x \le 450$.

(a) Graph the equation.

(b) How many cooktops must be produced to begin generating a profit?

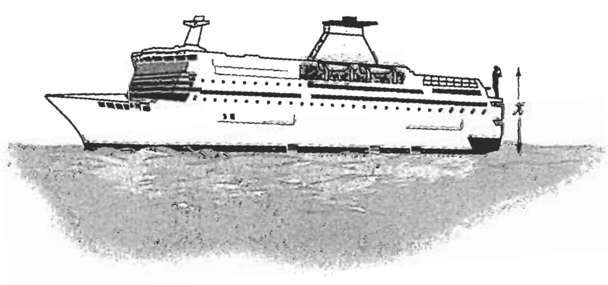
(c) For what range of values of x is the company's profit greater than $15,000?

59. How Far Can You See? If you stand on a ship in a calm sea, then your height x (in ft) above sea level is related to the farthest distance y (in mi) that you can see by the equation

$$y = \sqrt{1.5x + \left(\frac{x}{5280}\right)^2}$$

(a) Graph the equation for $0 \le x \le 100$.

(b) How high up do you have to be to be able to see 10 mi?



Discovery • Discussion

60. Equation Notation on Graphing Calculators When you enter the following equations into your calculator, how does what you see on the screen differ from the usual way of writing the equations? (Check your user's manual if you're not sure.)

(a) $y = |x|$

(b) $y = \sqrt[5]{x}$

(c) $y = \dfrac{x}{x - 1}$

(d) $y = x^3 + \sqrt[3]{x + 2}$

61. Enter Equations Carefully A student wishes to graph the equations

$$y = x^{1/3} \quad \text{and} \quad y = \frac{x}{x + 4}$$

on the same screen, so he enters the following information into his calculator:

```
Y1 = X^1/3        Y2 = X/X + 4
```

The calculator graphs two lines instead of the equations he wanted. What went wrong?

62. Algebraic and Graphical Solution Methods Write a short essay comparing the algebraic and graphical methods for solving equations. Make up your own examples to illustrate the advantages and disadvantages of each method.

63. How Many Solutions? This exercise deals with the family of equations

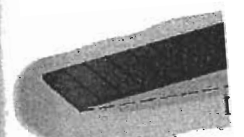
$$x^3 - 3x = k$$

(a) Draw the graphs of

$$y_1 = x^3 - 3x \quad \text{and} \quad y_2 = k$$

in the same viewing rectangle, in the cases $k = -4, -2, 0, 2$, and 4. How many solutions of the equation $x^3 - 3x = k$ are there in each case? Find the solutions correct to two decimals.

(b) For what ranges of values of k does the equation have one solution? two solutions? three solutions?



Slope of a

Slope =

Figure 1

where m and b are constants. When $h = 0$, we are given that $T = 20$, so

$$20 = m(0) + b$$
$$b = 20$$

Thus, we have

$$T = mh + 20$$

When $h = 1$, we have $T = 10$ and so

$$10 = m(1) + 20$$
$$m = 10 - 20 = -10$$

The required expression is

$$T = -10h + 20$$

(b) The graph is sketched in Figure 19. The slope is $m = -10°\text{C/km}$, and this represents the rate of change of temperature with respect to distance above the ground. So the temperature *decreases* 10°C per kilometer of height.

(c) At a height of $h = 2.5$ km, the temperature is

$$T = -10(2.5) + 20 = -25 + 20 = -5°\text{C}$$

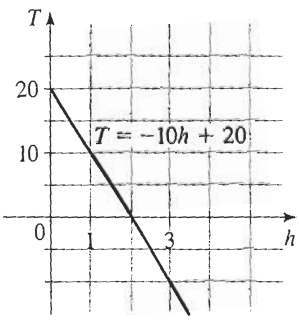


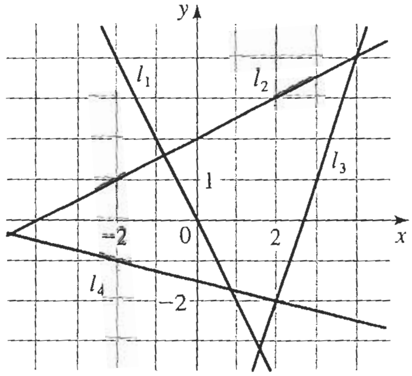
Figure 19

1.10 Exercises

1–8 ■ Find the slope of the line through P and Q.

1. $P(0,0)$, $Q(4,2)$
2. $P(0,0)$, $Q(2,-6)$
3. $P(2,2)$, $Q(-10,0)$
4. $P(1,2)$, $Q(3,3)$
5. $P(2,4)$, $Q(4,3)$
6. $P(2,-5)$, $Q(-4,3)$
7. $P(1,-3)$, $Q(-1,6)$
8. $P(-1,-4)$, $Q(6,0)$

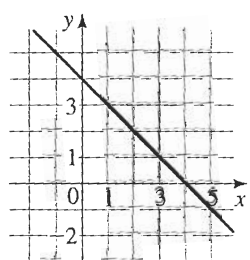
9. Find the slopes of the lines l_1, l_2, l_3, and l_4 in the figure below.



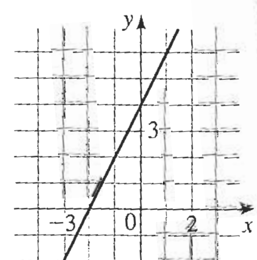
10. (a) Sketch lines through $(0,0)$ with slopes $1, 0, \frac{1}{2}, 2$, and -1.
 (b) Sketch lines through $(0,0)$ with slopes $\frac{1}{3}, \frac{1}{2}, -\frac{1}{3}$, and 3.

11–14 ■ Find an equation for the line whose graph is sketched.

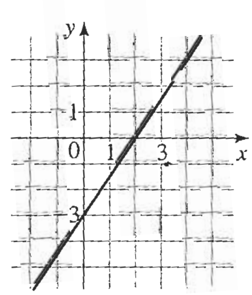
11.



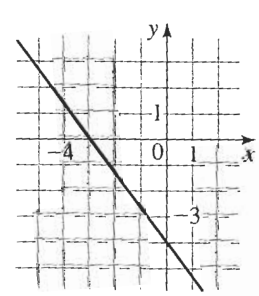
12.



13.



14.



15–34 ■ Find an equa
conditions.

15. Through $(2,3)$;
16. Through $(-2,4)$;
17. Through $(1,7)$;
18. Through $(-3,-5$
19. Through $(2,1)$ an
20. Through $(-1,-2$
21. Slope 3; y-interc
22. Slope $\frac{2}{5}$; y-interc
23. x-intercept 1; y-i
24. x-intercept -8;
25. Through $(4,5)$;
26. Through $(4,5)$;
27. Through $(1,-6)$;
28. y-intercept 6; pa
29. Through $(-1,2)$;
30. Through $(2,6)$;
31. Through $(-1,-2)$
 $2x + 5y + 8 = 0$
32. Through $(\frac{1}{2}, -\frac{2}{3})$;
33. Through $(1,7)$;
 $(2,5)$ and $(-2,1)$
34. Through $(-2,-11$
 through $(1,1)$ and
35. (a) Sketch the line
 point $(-2,1)$.
 (b) Find an equat
36. (a) Sketch the lin
 point $(4,-1)$
 (b) Find an equa

37–40 ■ Use a grap
lines in the same vie
common?

37. $y = -2x + b$
38. $y = mx - 3$ f
39. $y = m(x - 3)$
40. $y = 2 + m(x +$

15–34 ■ Find an equation of the line that satisfies the given conditions.

15. Through $(2, 3)$; slope 1
16. Through $(-2, 4)$; slope -1
17. Through $(1, 7)$; slope $\frac{2}{3}$
18. Through $(-3, -5)$; slope $-\frac{7}{2}$
19. Through $(2, 1)$ and $(1, 6)$
20. Through $(-1, -2)$ and $(4, 3)$
21. Slope 3; y-intercept -2
22. Slope $\frac{2}{5}$; y-intercept 4
23. x-intercept 1; y-intercept -3
24. x-intercept -8; y-intercept 6
25. Through $(4, 5)$; parallel to the x-axis
26. Through $(4, 5)$; parallel to the y-axis
27. Through $(1, -6)$; parallel to the line $x + 2y = 6$
28. y-intercept 6; parallel to the line $2x + 3y + 4 = 0$
29. Through $(-1, 2)$; parallel to the line $x = 5$
30. Through $(2, 6)$; perpendicular to the line $y = 1$
31. Through $(-1, -2)$; perpendicular to the line $2x + 5y + 8 = 0$
32. Through $(\frac{1}{2}, -\frac{2}{3})$; perpendicular to the line $4x - 8y = 1$
33. Through $(1, 7)$; parallel to the line passing through $(2, 5)$ and $(-2, 1)$
34. Through $(-2, -11)$; perpendicular to the line passing through $(1, 1)$ and $(5, -1)$

35. (a) Sketch the line with slope $\frac{3}{2}$ that passes through the point $(-2, 1)$.
    (b) Find an equation for this line.

36. (a) Sketch the line with slope -2 that passes through the point $(4, -1)$.
    (b) Find an equation for this line.

37–40 ■ Use a graphing device to graph the given family of lines in the same viewing rectangle. What do the lines have in common?

37. $y = -2x + b$ for $b = 0, \pm 1, \pm 3, \pm 6$
38. $y = mx - 3$ for $m = 0, \pm 0.25, \pm 0.75, \pm 1.5$
39. $y = m(x - 3)$ for $m = 0, \pm 0.25, \pm 0.75, \pm 1.5$
40. $y = 2 + m(x + 3)$ for $m = 0, \pm 0.5, \pm 1, \pm 2, \pm 6$

41–52 ■ Find the slope and y-intercept of the line and draw its graph.

41. $x + y = 3$
42. $3x - 2y = 12$
43. $x + 3y = 0$
44. $2x - 5y = 0$
45. $\frac{1}{2}x - \frac{1}{3}y + 1 = 0$
46. $-3x - 5y + 30 = 0$
47. $y = 4$
48. $4y + 8 = 0$
49. $3x - 4y = 12$
50. $x = -5$
51. $3x + 4y - 1 = 0$
52. $4x + 5y = 10$

53. Use slopes to show that $A(1, 1)$, $B(7, 4)$, $C(5, 10)$, and $D(-1, 7)$ are vertices of a parallelogram.

54. Use slopes to show that $A(-3, -1)$, $B(3, 3)$, and $C(-9, 8)$ are vertices of a right triangle.

55. Use slopes to show that $A(1, 1)$, $B(11, 3)$, $C(10, 8)$, and $D(0, 6)$ are vertices of a rectangle.

56. Use slopes to determine whether the given points are collinear (lie on a line).
    (a) $(1, 1), (3, 9), (6, 21)$
    (b) $(-1, 3), (1, 7), (4, 15)$

57. Find an equation of the perpendicular bisector of the line segment joining the points $A(1, 4)$ and $B(7, -2)$.

58. Find the area of the triangle formed by the coordinate axes and the line

$$2y + 3x - 6 = 0$$

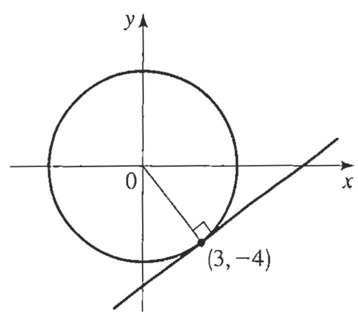
59. (a) Show that if the x- and y-intercepts of a line are nonzero numbers a and b, then the equation of the line can be written in the form

$$\frac{x}{a} + \frac{y}{b} = 1$$

This is called the **two-intercept form** of the equation of a line.

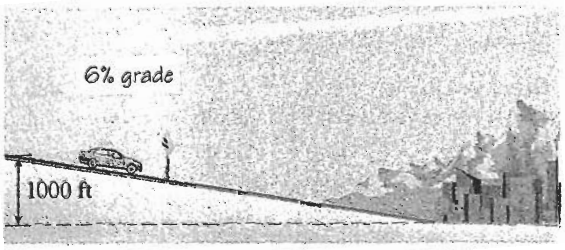
    (b) Use part (a) to find an equation of the line whose x-intercept is 6 and whose y-intercept is -8.

60. (a) Find an equation for the line tangent to the circle $x^2 + y^2 = 25$ at the point $(3, -4)$. (See the figure.)
    (b) At what other point on the circle will a tangent line be parallel to the tangent line in part (a)?



Applications

61. **Grade of a Road** West of Albuquerque, New Mexico, Route 40 eastbound is straight and makes a steep descent toward the city. The highway has a 6% grade, which means that its slope is $-\frac{6}{100}$. Driving on this road you notice from elevation signs that you have descended a distance of 1000 ft. What is the change in your horizontal distance?



62. **Global Warming** Some scientists believe that the average surface temperature of the world has been rising steadily. The average surface temperature is given by

$$T = 0.02t + 8.50$$

where T is temperature in °C and t is years since 1900.

(a) What do the slope and T-intercept represent?

(b) Use the equation to predict the average global surface temperature in 2100.

63. **Drug Dosages** If the recommended adult dosage for a drug is D (in mg), then to determine the appropriate dosage c for a child of age a, pharmacists use the equation

$$c = 0.0417D(a + 1)$$

Suppose the dosage for an adult is 200 mg.

(a) Find the slope. What does it represent?

(b) What is the dosage for a newborn?

64. **Flea Market** The manager of a weekend flea market knows from past experience that if she charges x dollars for a rental space at the flea market, then the number y of spaces she can rent is given by the equation $y = 200 - 4x$.

(a) Sketch a graph of this linear equation. (Remember that the rental charge per space and the number of spaces rented must both be nonnegative quantities.)

(b) What do the slope, the y-intercept, and the x-intercept of the graph represent?

. **Production Cost** A small-appliance manufacturer finds that if he produces x toaster ovens in a month his production cost is given by the equation

$$y = 6x + 3000$$

(where y is measured in dollars).

(a) Sketch a graph of this linear equation.

(b) What do the slope and y-intercept of the graph represent?

66. **Temperature Scales** The relationship between the Fahrenheit (F) and Celsius (C) temperature scales is given by the equation $F = \frac{9}{5}C + 32$.

(a) Complete the table to compare the two scales at the given values.

(b) Find the temperature at which the scales agree. [*Hint:* Suppose that a is the temperature at which the scales agree. Set $F = a$ and $C = a$. Then solve for a.]

C	F
−30°	
−20°	
−10°	
0°	
	50°
	68°
	86°

67. **Crickets and Temperature** Biologists have observed that the chirping rate of crickets of a certain species is related to temperature, and the relationship appears to be very nearly linear. A cricket produces 120 chirps per minute at 70°F and 168 chirps per minute at 80°F.

(a) Find the linear equation that relates the temperature t and the number of chirps per minute n.

(b) If the crickets are chirping at 150 chirps per minute, estimate the temperature.

68. **Depreciation** A small business buys a computer for \$4000. After 4 years the value of the computer is expected to be \$200. For accounting purposes, the business uses *linear depreciation* to assess the value of the computer at a given time. This means that if V is the value of the computer at time t, then a linear equation is used to relate V and t.

(a) Find a linear equation that relates V and t.

(b) Sketch a graph of this linear equation.

(c) What do the slope and V-intercept of the graph represent?

(d) Find the depreciated value of the computer 3 years from the date of purchase.

69. **Pressure and Depth** At the surface of the ocean, the water pressure is the same as the air pressure above the water, 15 lb/in^2. Below the surface, the water pressure increases by 4.34 lb/in^2 for every 10 ft of descent.

(a) Find an equation for the relationship between pressure and depth below the ocean surface.

(b) Sketch a graph of this linear equation.

(c) What do the slope and y-intercept of the graph represent?

(d) At what depth is the

70. **Distance, Speed, and T**
Detroit at 2:00 P.M. and d
west on I-90. They pass
2:50 P.M.

(a) Express the distance
elapsed.

(b) Draw the graph of th

(c) What is the slope of

71. **Cost of Driving** The r
depends on the number
May her driving cost wa
cost was \$460 for 800 m

Mathematical models are
more detail in *Focus on M*
which begins on page 239