Let $f$ be a function defined on some open interval that contains the number $a$, except possibly at $a$ itself. Then we say that the limit of $f(x)$ as $x$ approaches $a$ is $L$, and we write
\[ \lim_{x \to a} f(x) = L \]
if for every number $\epsilon > 0$ there is a corresponding number $\delta > 0$ such that
\[ |f(x) - L| < \epsilon \quad \text{whenever} \quad 0 < |x - a| < \delta \]

Now let's put this definition to work.

**Prove that** $\lim_{x \to 3} (7x + 2) = 23$.

(First we must guess a value for $\delta$.)

Let $\epsilon$ be any positive number. We want to find a number $\delta$ such that
\[ |(7x + 2) - 23| < \epsilon \quad \text{whenever} \quad 0 < |x - 3| < \delta. \]
Now, $|(7x + 2) - 23| = |7x - 21| = |7(x - 3)| = 7| x - 3 |$. So, we want whenever
\[ |x - 3| < \epsilon/7 \quad \text{whenever} \quad 0 < |x - 3| < \delta. \]
Choose $\delta = \epsilon/7$.

(Now, we are ready to write a proof that is to show this number $\delta$ works.)

**Proof:** Given $\epsilon > 0$, choose $\delta = \epsilon/7$. Then, whenever $0 < |x - 3| < \delta$,
\[ |(7x + 2) - 23| = |7x - 21| = |7(x - 3)| = 7| x - 3 | = 7| x - 3 | < 7\delta = 7 \cdot \frac{\epsilon}{7} = \epsilon. \]
Thus, $|(7x + 2) - 23| < \epsilon$ whenever $0 < |x - 3| < \delta$.
Therefore, $\lim_{x \to 3} (7x + 2) = 23$. 

Go to the next page and you practice.
Prove that \( \lim_{x \to 2} (5x + 8) = 18. \)

(First we must guess a value for \( \delta \).)

Let \( \epsilon \) be any positive number. We want to find a number \( \delta \) such that

\[
\text{______________ whenever ______________.}
\]

Now, \( \text{________________________.} \)

So, we want __________ whenever __________

that is ______________ whenever ______________.

Choose \( \delta = _____ \).

(Now, we are ready to write a proof that is to show this number \( \delta \) works.)

\textit{Proof} : Given \( \epsilon > 0 \), choose \( \delta = _____ \). Then, whenever ____________,

\[
\text{___________________________________________}
\]

\[
\text{___________________________________________}
\]

Thus, ______________ whenever ______________.

Therefore, ______________.

Let's do one more.

Prove that \( \lim_{x \to -1} (3x - 7) = -10. \)