Group Work 1, Section 3.10 Theme on Four Variations

Consider the following four functions:

$$f(x) = -1 + \sqrt{2x+1}$$

$$g\left(x\right) = x^3 + x$$

$$h\left(x\right) = \tan^2 x + x$$

$$j\left(x\right) =\sin x$$

1. Find the linearizations of f, g, h, and j near x = 0.

2. Something interesting should have happened in Problem 1. What was special about those four particular functions that made the result unusual?

3. For each function, note how long it takes before its linear approximation fails badly. (For the purposes of this discussion, we can define "failing badly" as deviating from its actual value by more than $\frac{1}{2}$). For which function is the approximation best? For which is it worst? Explain why this is so.