College Algebra
Workshop 7

Quadratic Functions

Simplify each of the following expressions.

1. $\dfrac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-2)}}{2(1)}$ **2.** $\dfrac{5 \pm \sqrt{25-16}}{2}$

3. $\dfrac{-6 \pm \sqrt{18}}{4}$ **4** $\dfrac{9 \pm \sqrt{45}}{6}$

For each of the following functions: (i) find the y-intercept; (ii) find the x-intercepts; (iii) find the coordinates of the vertex of the graph of the function; (iv) sketch a good graph of the function; (v) find the range; (vi) find the sets of upper inputs and lower inputs; (vii) find the intervals where the function outputs are increasing and the intervals where the function outputs are decreasing. Give exact answers.

5. $f(x) = x^2 + 3x - 5$

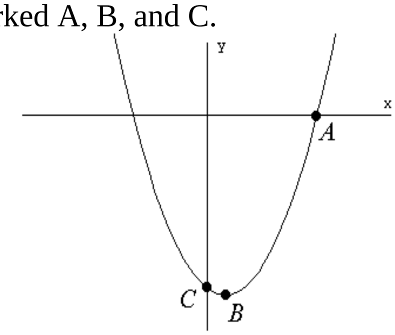
6. $g(x) = -x^2 + 5x + 2$

Use your results from Exercises 5–6 to solve the following. Give exact answers.

7. $x^2 + 3x - 5 < 0$

8. $2 < x^2 - 5x$

9. A graph of the function $f(x) = 2x^2 - 2x - 12$ is shown below. Analytically find the coordinates of the points marked A, B, and C.



Graph of $f(x) = 2x^2 - 2x - 12$

10. Managers of apartment complexes have long been aware of the following phenomenon: it is sometimes better to charge rents that are so high that not all the apartments in the complex will be leased. In other words, rather than lowering rents to attract more tenants, it may be better to keep rents higher, even if this means some apartments will remain vacant. Let's explore a particular example of this situation to help understand why this phenomenon occurs. Marigold manages the Jacksonville Arms apartment complex in Jacksonville, Florida. Marigold knows from past experience that at an average rent of $\$400$ per month, all 200 units in the complex will be leased. The revenue earned in a month at this rent is then $(400)(200)$ = $\$80,000$. However, for each $\$10$ increase in the average rent, about 2 tenants will be lost. Of course, Marigold wants to know how much she should raise the rent, if any, above an average of $\$400$ in order to maximize monthly revenue. Let x be the number of $\$10$ rent increases Marigold charges above an average of $\$400$. For instance, if the average rent is $\$450$, then $x = 5$, and the number of tenants will decrease to $200 - 5(2) = 190$. The monthly revenue would then be $(450)(190) = \$85,500$.

a. Write an expression containing x that stands for the average rent at the Jacksonville Arms (after Marigold implements the x rent increases).

b. Write an expression containing x that stands for the number of tenants at the Jacksonville Arms (again after Marigold implements the x rent increases).

c. Use parts a and b to write the symbol rule for a function $R(x)$ that outputs the monthly revenue for the Jacksonville Arms, where the number x of $\$10$ rent increases is the input. Show that R is a parabola by writing the symbol rule for R in explicit form and identifying the coefficients a, b, and c.

d. Write the abstract domain of the function R using interval notation.

e. Sketch a good graph of the function R from part d. Use the graph to estimate the x-intercepts of the function and the coordinates of the vertex.

f. Use your graph from part a to write the application domain of the function R. Use set-builder notation.

g. Analytically find the coordinates of the vertex of the function R.

h. How many $\$10$ rent increases above $\$400$ should Marigold implement to maximize monthly revenue? What is the maximum monthly revenue?