Test 2 2405 Name ______________

1. (3 pts) **Find examples of** sets A, B and C that such that $A-C=B-C$ but $A \neq B$.

Let $A=\{1,2\}$
$B=\{1,3\}$
$C=\{2,3\}$.
Then $A-C=\{1\}=B-C$ but $A \neq B$

2. (9 pts) Let i be a positive integer and let $A_i = \{i, i+1, i+2, \ldots, 2i\}$. Assume n is greater than 1. **List the element** of the following sets.

$A_2=\{2,3,4\}$
$A_3=\{3,4,5,6\}$
$A_4=\{4,5,6,7,8\}$
$A_5=\{5,6,7,8,9,10\}$

i. $A_3 = \{3,4,5,6\}$

ii. $\bigcap_{i=2}^{4} A_i = \{4\}$

iii. $\bigcap_{i=2}^{500} A_i = \emptyset$

iv. $\bigcup_{i=1}^{4} A_i = \{1,2,3,4,5,6,7,8\}$

v. $\bigcup_{i=1}^{500} A_i = \{1,2,3,\ldots,500,501,\ldots,1000\}$

3. (8 pts) **Prove** the set identity $\overline{A \cup (\overline{B \cap C})} = C \cap (B \cap \overline{A})$ for any sets A, B and C using set identities (provide reasons at each step).

$$\overline{A \cup (\overline{B \cap C})} = \overline{A} \cap \overline{\overline{(B \cap C)}}$$
$$= \overline{A} \cap (B \cap C)$$
$$= (B \cap C) \cap \overline{A}$$
$$= (C \cap B) \cap \overline{A}$$

Must prove $x \in A \cup B \to x \in A \cup B \cup C$

4. (5 points) Consider the proposition that $A \cup B \subseteq A \cup B \cup C$. For each of the following, indicate whether it is the first line of a *direct proof* of the proposition, *indirect proof* the proposition, *proof by contradiction* the proposition, or *none of these*. **Note**: this problem assumes you know the definition of subset.

a. Let $x \in A \cup B$. direct

b. Let $x \in A \cup B \cup C$. NONE

c. Suppose $x \in A \cup B$ and $x \in A \cup B \cup C$. NONE

d. Suppose $x \in A \cup B$ and $x \notin A \cup B \cup C$. Contradiction

e. Suppose $x \notin A \cup B \cup C$. Indirect

5. (11 pts) Let ***Z*** be the set of integers, and ***Z***$^+$ be the set of positive integers. Let ***E*** be the set of even integers, and ***E***$^+$ be the set of positive even integers. Let ***S*** be the set of all finite length bit strings.

i. Let $f: Z \to Z^+$, where $f(x) = x^2$. Explain why this assignment of integers to the positive integers is NOT a one-to-one function.

$f(1) = 1^2 = (-1)^2 = f(-1)$

but $1 \neq -1$

ii. Let $f: S \to E^+ \cup \{1\}$, where if *x* is a bit string of length *n*, then $f(x) = 2^n$. Use this function to answer parts a – d.

a. Evaluate $f(001)$ = 2^3

b. Explain why this function is NOT onto $E^+ \cup \{1\}$.

6 is even but not equal to 2^n for any pos int. n.

c. Explain why this function is NOT one-to-one.

$f(001) = 2^3 = f(100)$

but 001 is not 100

d. TRUE or FALSE: This function is invertible.

b/c is if not 1-1, not invertible

6. (8 pts) Let f be the real value function $f(x) = \lfloor x/3 \rfloor$.

i. Evaluate $f(1) =$ <u>0</u>

ii. Evaluate $f(-1.4) =$ <u>-1</u> $\lfloor \frac{-1.4}{3} \rfloor$

iii. The image of 7 is <u>2</u> $\lfloor \frac{7}{3} \rfloor$

iv. All pre-images of 2 are $\{x \mid 6 \le x < 9\}$

v. Let S be the set of real numbers strictly between 0 and 6 (i.e. in the interval (0,6)). Find the image of the set S, i.e. find $f(S)$.

for $0 < x < 3$ $f(x) = 0$
$3 \le x < 6$ $f(x) = 1$

vi. Let $S = \{1, 2\}$. Find the pre-image of the set S, i.e. find $f^{-1}(S)$.

Since $3 \le x < 6$ $f(x) = 1$ and
$6 \le x < 9$ $f(x) = 2$, $f^{-1}(S) = \{x \mid 3 \le x < 9\}$

7. (6 pts) Let $f : A \to B$ and $g : B \to C$ both be onto. Fill in the blanks for the proof that $g \circ f : A \to C$ is onto.

Proof. Let $y \in C$. Since <u>g is onto</u>, there exists an x in B such that $g(x)=y$. And since <u>f is onto</u>, there exists a z in A such that $f(z) = x$. Now by definition of composition and by substitution we see that $(g \circ f)(z) = g(f(z)) =$ <u>y</u>. This completes our proof that $g \circ f$ is onto, since we have proven that for any $y \in C$ there exists a <u>$z \in A$</u> such that $(g \circ f)(z)=y$. □

8. (7 pts) Let $f : R \to (-1,0]$ such that $f(x) = \lfloor x \rfloor - x$.

i. Evaluate $f(2)=$ $\lfloor 2 \rfloor - 2 = \boxed{0}$

ii. Evaluate $f(2.5)=$ $\lfloor 2.5 \rfloor - 2.5 = 2 - 2.5 = \boxed{-0.5}$

iii. Evaluate $f(-2.5)=$ $\lfloor -2.5 \rfloor - (-2.5) = -3 + 2.5 = \boxed{-0.5}$

iv. Evaluate $f(1.75)=$ $\lfloor 1.75 \rfloor - 1.75 = 1 - 1.75 = \boxed{-0.75}$

v. Evaluate $f(-1.75)=$ $\lfloor -1.75 \rfloor - (-1.75) = -2 + 1.75 = -0.25$

vi. Find an x such that $f(x)= -0.367$ $\lfloor +1.367 \rfloor - (1.367) = 1 - 1.367$

$\boxed{x = 1.367}$

vii. TRUE or FALSE: f is onto.

9. (10 pts) Suppose $f : R \to R$ that has the rule $f(n) = 3n - 7$.

(a) **Prove** that this function is one-to-one.

proof: Let $x, y \in \mathbb{R}$.
Assume $f(x) = f(y)$,

$$\left.\begin{aligned} 3x - 7 &= 3y - 7 \end{aligned}\right\} \text{substitution}$$

$$\left.\begin{aligned} 3x &= 3y \\ x &= y \end{aligned}\right\} \text{simple algebra}$$

Thus $\forall x \forall y\ f(x) = f(y) \rightarrow x = y$, i.e. f is 1-1. $\square$

(b) **Prove** that this function is onto R.

proof: Let $y \in \mathbb{R}$. Since $\frac{y+7}{3} \in \mathbb{R}$ and
$f\left(\frac{y+7}{3}\right) = 3\left(\frac{y+7}{3}\right) - 7 = y$, f is onto $\mathbb{R}$.
$\square$

10. (6 pts) **Sequences**

i. **List the first 4 terms** of the sequence $\left\{\dfrac{2}{i+3}\right\}_{i=0}$

$\frac{2}{3}, \frac{2}{4}, \frac{2}{5}, \frac{2}{6}$

ii. **List the first 4 terms** of the arithmetic sequence determined by $a = 2$ (initial value) and $d = 5$ (the common difference).

2, 7, 12, 17

iii. **List the first 4 terms** of the geometric sequence determined by $a = 2$ (initial value) and $r = 5$ (the common ratio).

2, 10, 50, 250

11 ~~8~~. (6 pts) **Sequence formulas**

a) For the list of integers provided **find a formula** that generates a sequence that begins with the list of integers. $\frac{1}{3}, \frac{2}{3^2}, \frac{3}{3^3}, \frac{4}{3^4}, \frac{5}{3^5}, \ldots$

$a_n = \frac{n}{3^n}$

b) **Provide a formula** that generates a sequence that begins with the list.

8, 98, 998, 9998, 99998, …

$10-2,\ 100-2,\ 1000-2, \ldots$

$\boxed{a_n = 10^n - 2}$

c) **Provide a formula** that generates a sequence that begins with the list.

8, 13, 18, 23, 28, …

5 5 5 5

$\boxed{a_n = 8 + 5(n-1)}$

12 ~~9~~. (6 pts) **Summations**

i. Evaluate the sum $\sum_{k=0}^{5} 3k = 3\cdot 0 + 3\cdot 1 + 3\cdot 2 + 3\cdot 3 + 3\cdot 4 + 3\cdot 5 = 45$

ii. Evaluate the sum $\sum_{k=0}^{5} 3 = 3+3+3+3+3+3 = 3\cdot 6 = 18$

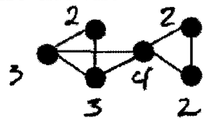
iii. Evaluate the product $\prod_{k=0}^{5} 3 = 3\cdot 3\cdot 3\cdot 3\cdot 3\cdot 3 = 3^6$

10. (2 points) Express $\sum_{k=0}^{98}(k+3)$ as a single equivalent summation whose lower limit is 2. That is find the upper limit and the argument so that $\sum_{k=0}^{98}(k+3) = \sum_{j=2}^{100}(j+1)$.

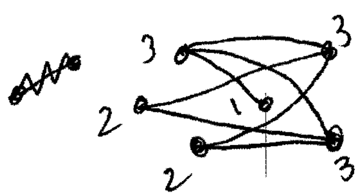
$j = k+2$

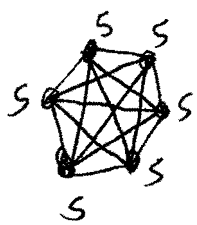
$j+1 = k+3$

13 ~~11~~. (15 pts) Determine the indicated graph parameters for the indicated graphs.

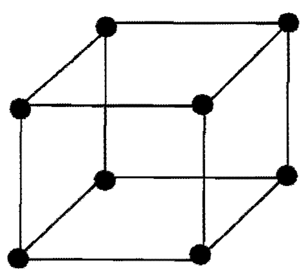
Let G be the graph  and use it to fill in the values in the first two rows of the table.

Graph	Number of vertices	Degree sequence	maximum degree	minimum degree	Number of edges
G	6	2,2,2,3,3,4	4	2	8
Complement of G	6	3,3,3,2,2,1	3	1	7
K_6, the complete graph on 6 vertices	6	5,5,5,5,5,5	5	5	15





14 ~~12~~. (3 pts) Recall that the graph called the n-Cube is the graph whose vertex set is the set of 2^n bitstrings of length n, with two vertices adjacent if and only if their respective bitstring representations differ in exactly one position. Below is the drawing of the 3-cube. Label the vertices of the drawing so that it is clear that this is the 3-cube.



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