Binomial Coefficients

Terminology: The number $\binom{n}{r}$ is also called a **binomial coefficient** because they occur as coefficients in the expansion of powers of binomial expressions such as $(a+b)^n$.

Theorem (Pascal's Identity) Let *n* and *k* be positive integers with

$n \geq k$. Then $\binom{n+1}{k} = \binom{n}{k-1} + \binom{n}{k}$.

Proof.

Using Pascal's Identity we can construct Pascal's Triangle

Theorem (The Binomial Theorem) Let x and y be variables, and let n be a positive integer. Then

$$(x+y)^n = \sum_{j=0}^{n} C(n, j) x^{n-j} y^j$$

$$= \binom{n}{0} x^n + \binom{n}{1} x^{n-1} y + \binom{n}{2} x^{n-2} y^2 + \cdots + \binom{n}{n-1} x^1 y^{n-1} + \binom{n}{n} y^n.$$

Example: Find the expansion $(x+y)^6$

Example: Find the coefficient of xy^5 in the expansion of $(2x+y)^7$