Homework 1 Name ______________________________
Due February $4^{th}$ at the beginning of class.

**1.** Write the contrapositive a "if $n$ is an even integer, then $n^2$ is an even integer."

**2.** Write the converse a "if $n$ is an even integer, then $n^2$ is an even integer."

**3.** Write the contrapositive a "if $n^2$ is an even integer, then $n$ is an even integer."

**4.** Write the converse a "if $n^2$ is an even integer, then $n$ is an even integer."

**5.** The negation of an implication.

**a.** In part b. of this exercise we will prove the statement $\neg(p \rightarrow q) \equiv p \wedge \neg q$, but first fill in the following blanks with one of the words *negation*, *conjunction, implication, hypothesis* or *conclusion* in order to describe the significance of the equivalence $\neg(p \rightarrow q) \equiv p \wedge \neg q$.

*The negation of an* ____________________ *is equivalent to the* ____________________ *of the hypothesis and the* ________________ *of the conclusion of the hypothesis.*

**b.** Provide the reasons (names of logical equivalences) for the proof.

$\neg(p \rightarrow q) \equiv \neg(\neg p \vee q)$ by ______________________

$\equiv \neg\neg p \wedge \neg q$ by ______________________

$\equiv p \wedge \neg q$ by ______________________

**c.** Let $p$ denote the statement that "$n$ is an even integer", and

let $q$ be the statement that "$n^2$ is an even integer".

Using the above English meaning of $p$ and $q$, write the English equivalent of the statement $\neg(p \rightarrow q) \equiv p \wedge \neg q$.

**d.** Write the English equivalent of the negation of the statement "if *n* is divisible by *3*, then $n^2$ is divisible by *3*" in the two different ways proposed in part c.

i.

ii.

**e.** Write the English equivalent of the contraposition of the statement "if *n* is divisible by *3*, then $n^2$ is divisible by *3*" in the two different ways proposed in part c.

**f.** Prove (using the laws of logic) that the following proposition is a tautology (give the reasons at each step). $\neg(p \rightarrow q) \vee \neg(p \wedge \neg q)$

**g.** If the truth-value of $p \wedge \neg q$ is false, then what is the truth-value of $(p \rightarrow q)$? Explain your reasoning.