Test 3

Spring 2006 students, this is the first semester that I cover Ch 8, so I have no sample questions. Do the practice homework, know the definitions and the theorems that we covered in Ch 8.

Your test 3 is over Graph Theory, Integers and Division, Sequences and Summations, and mathematical induction.

In a couple of places in the test you may need the exactly wording of the Division Algorithm Theorem, so I am providing it for you.

Division Algorithm: Let *a* be an integer and *d* a positive integer. Then there are ***unique*** integers *q* and *r*, with $0 \le r < d$, such that $a = dq + r$.

I. Integer and Division

1.
   - ii. Find GCD($2^4 \cdot 5^4 \cdot 7^3 \cdot 17,\ 2^4 \cdot 5^3 \cdot 7^2 \cdot 17^3 \cdot 19$)
   - iii. Find LCM($2^4 \cdot 5^4 \cdot 7^3 \cdot 17,\ 2^4 \cdot 5^3 \cdot 7^2 \cdot 17^3 \cdot 19$)

2. Use the **Euclidean Algorithm** to find the GCD of 168 and 456.

3. True/False.
   - i. $13 \equiv 3 \bmod 5$
   - ii. $0 \equiv 5 \bmod 5$
   - iii. $-1 \equiv 19 \bmod 5$
   - iv. $-1 \equiv -16 \bmod 4$

5. Let $R_1 = \{n \mid n \equiv 1 (\bmod 3)\}$ and $R_2 = \{n \mid n \equiv 2 (\bmod 3)\}$.

   Use a proof by contradiction to prove that $R_1 \cap R_2 = \varnothing$.

   Hint: You will need the Division Algorithm Theorem to reach a contradiction.

6. Store the following student ID numbers in memory locations 0-9 using a hashing function f(s) = s(mod 10). Explain how you managed collisions.

658779
648551
658881
658772
657991
648552
657102

657112
657122
657100

II. Proofs

1. Disprove that $\lfloor x-y \rfloor = \lfloor x \rfloor - \lfloor y \rfloor$.
2. Consider: $\forall x \in Z$, if $3x+1$ is odd, then $3x$ is even.
    a. Give a direct proof.
    b. Give an indirect proof.
    c. Give a proof by contradiction.

III. More Proofs

1. Prove that if $3 \mid n^2$, then $3 \mid n$
2. Prove that $\sqrt{3}$ is irrational.

Hint: You must do # 1 before # 2, since part 2 will be incomplete without it.

IV. Mathematical Induction

1. State the Theorem of the Principle of Mathematical Induction.
2. Use the principal of math induction to prove **one** of the following (i or ii exclusively).

    i. $\displaystyle\sum_{i=1}^{n} \frac{i}{2^i} = \frac{2^{n+1} - 2 - n}{2^{\mathrm{n}}} \quad \forall n \geq 1$

    ii. $\displaystyle\sum_{i=1}^{n} 3^i = \frac{3(3^n - 1)}{2} \quad \forall n \geq 1$

V. Sequences and Summation

1. Write the following in sigma notation: $2a + 2a^2 + 2a^3 + 2a^4 + \ldots + 2a^{10}$.
2. List the first six terms of the sequence $\left\{\dfrac{2^i}{i}\right\}$.
3. Let S = {1, 5, 8, 9}. Evaluate the following $\displaystyle\sum_{i \in S} i + 2$.

4. Let S = {1, 5, 8, 9}. Evaluate the following $\sum_{i \in S} (i+2)$.

5. Express $\sum_{k=0}^{98} (k+1)$ as a single equivalent summation whose lower limit is 2.

6. For the list of integers provided a fomula that generates a sequence that begins with the list.

(a) -2, 3, 8, 13, 18, 23, 28, ….

(b) 9, 99, 999, 9999, 99999, …