

# Sample Questions for test 2, but please look at the notes and homework as this is only a sample (not a review)

## I. Methods of Proofs

1. Prove
  - (a) Indirectly if  $3n+2$  is odd, then  $n$  is odd.
  - (b) By contradiction that if  $3n+2$  is odd, then  $n$  is odd.
2. Prove that the difference of two odd integers is an even integer.

## II. Functions

- 1) True or False?

Let  $R$  be the set of real numbers, and  $R^+$  be the set of positive real numbers. Let  $Z$  be the set of integers, and  $Z^+$  be the set of positive integers. Let  $E$  be the set of even integers, and  $E^+$  be the set of positive even integers. Let  $S$  be the set of all finite length bit strings.

  - i. **True or False:** Let  $f : R \rightarrow R$ , where  $f(x) = x^2$ . This assignment of the integers to the integers is a function.
  - ii. **True or False:** Let  $f : Z^+ \rightarrow Z^+$ , where  $f(x) = x^2$ . This assignment of positive integers to the positive integers is a one-to-one function.
  - iii. **True or False:** Let  $f : S \rightarrow E \cup \{1\}$ , where if  $x$  is a bit string of length  $n$  such that  $f(x) = 2^n$ . This function is one-to-one.
  - iv. **True or False:** Let  $f : S \rightarrow E^+ \cup \{1\}$ , where if  $x$  is a bit string of length  $n$  such that  $f(x) = 2^n$ . This function is onto  $E^+ \cup \{1\}$ .
  - v. **True or False:** Let  $f : R^+ \rightarrow Z$ , where  $f(x) = \left\lfloor \frac{x}{2} \right\rfloor$ . This function is one-to-one.
  - vi. **True or False:** Sunday is the 948<sup>th</sup> day from Thursday.
- 2) Suppose  $f : R \rightarrow R$  that has the rule  $f(x) = 5n - 2$ .
  - (a) **Prove** that this function is one-to-one.
  - (b) **Prove** that this function is onto  $R$ .

### III. Sets

1. Suppose  $A = \{a, b, c\}$ . Mark each of the following TRUE or FALSE.

(a)  $\{b, c\} \in P(A)$

(b)  $\{\emptyset\} \in P(A)$

(c)  $\emptyset \in P(A)$

(d)  $\{b, c\} \subseteq P(A)$

(e)  $|P(A)| = 9$

(f)  $A \oplus (B \oplus C) = (A \oplus B) \oplus C$

2. Prove that for any set  $A$ ,  $\emptyset \subseteq A$ .

3. Let  $|U| = 200$ ,  $|A| = 97$ ,  $|B| = 135$ , and  $|A \cap B| = 32$ . Find the cardinality of  $|A \cup B|$ .

4. Prove that

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|.$$

### IV. Set Operations

1. Determine the union and intersection.

(b)  $\bigcap_{i=1}^n \{i, i+1, \dots, n\}$

(c)  $\bigcup_{i=1}^n \{i, i+1, \dots, n\}$

2. Disprove  $A \times B = B \times A$ .

3. Let  $A = \{1, 2, 3, 4\}$ .

(a)  $|P(A)| =$

(b)  $|A \times A| =$

(c)  $|P(A \times A)| =$

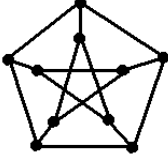
(d)  $|P(A) \times P(A)| =$

(e)  $|A \times A \times A| =$

4. **Prove or disprove** that  $\overline{(A - B)} = \overline{A} \cup B$  whenever  $A$  and  $B$  are sets.

5. Prove (no Venn Diagram or membership table proofs for this):

$$A - (B \cap C) = (A - B) \cup (A - C).$$

	List Degree sequence	Num. of edges.	Max. degree	Max. degree	Sum of degrees	True or False The graph has $P_3$ as a subgraph	True or False The graph has $P_3$ as an induced subgraph
$K_5$							
$\overline{K_{10}}$							
$P_7$							
							

Please see notes and homework on all sections, since this is only a small sample.