This is **not** a review sheet; you should review all practice homework (the practice and handout) problems. This is simply a **sample** of logic questions that I have asked in previous semesters. I cannot ask the questions on this page, thus I strongly recommend that you rely on the homework more than this sheet. However, I thought it might be useful for you to know the types of questions and difficulty level.

1. Construct a truth table for $p \rightarrow (p \vee q)$.

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2. Construct a truth table for $\neg(r \rightarrow \neg q) \vee (p \wedge \neg r)$.

3. Write each of the following in *"if p, then q"* form in English.
   (a) Horses fly whenever the sheep roam.

   (b) Sheep roam if horses fly.

4. Complete the following truth table and answer the true and false questions afterwards.
   (a)

p	q	$p \rightarrow q$	$\neg (p \rightarrow q)$	$q \rightarrow p$	$\neg p$	$\neg q$	$\neg p \rightarrow \neg q$	$\neg q \rightarrow \neg p$
T	T							
T	F							
F	T							
F	F							

   (b) True or False: an implication and its converse are logically equivalent.
   (c) True or False: an implication and its inverse are logically equivalent.
   (d) True or False: an implication and its contrapositive are logically equivalent.
   (e) True or False: the inverse of an implication and its converse are logically equivalent.

5. True or False:
   (i) True or False: $p \to q \equiv p \leftrightarrow q$
   (ii) True or False: $F \to q \equiv T$
   (iii) True or False: $p \to q \equiv p \oplus q$
   (iv) True or False: $p \to q \equiv \neg p \vee q$
   (v) True or False: $p \to q \equiv p \wedge \neg q$
   (vi) True or False: $\neg(p \to q) \equiv \neg p \to \neg q)$

6. For each of the following logical equivalences state the name of the logical equivalence that make it true.

   (a) $(p \vee q) \wedge F \equiv F$ ____________________

   (b) $(p \vee q) \wedge (p \vee q) \equiv p \vee q$ ____________________

   (c) $(p \vee q) \vee F \equiv p \vee q$ ____________________

   (d) $(p \to q) \vee \neg(p \to q) \equiv T$ ____________________

   (e) $\neg[(p \vee q) \wedge r] \equiv \neg(p \vee q) \vee \neg r$ ____________________

7. Prove that $p \to (p \vee q)$ is a tautology using propositional equivalence and the laws of logic, (give reasons at each step.)

8. A proof that $(p \oplus q) \wedge (p \wedge q) \equiv F$ is given below, the exercise is to provide the reasons i.e. the names of equivalence laws of logic use at each step.

Proof:

$(p \oplus q) \wedge (p \wedge q) \equiv$	
$[(p \vee q) \wedge \neg(p \wedge q)] \wedge (p \wedge q) \equiv$	
$(p \vee q) \wedge [\neg(p \wedge q) \wedge (p \wedge q)] \equiv$	
$(p \vee q) \wedge [(p \wedge q) \wedge \neg (p \wedge q)] \equiv$	
$(p \vee q) \wedge \mathrm{F} \equiv$	
F	

Done.

9. Prove that $(q \wedge (p \rightarrow \neg q)) \rightarrow \neg p$ is a tautology using propositional equivalence and the laws of logic, (give reasons at each step.)

10. Let *P(m,n)* be the statement $n \geq m$, where the universe of discourse for *m* and *n* is the set of nonnegative integers. Circle the truth value for each of the following:
(a) Write the following proposition in English. $\forall n \ P(0,n)$

(b) Determine the truth value of: $\forall n \ P(0,n)$. True or False.

(c) Write the following proposition in English . $\neg \forall n \ P(0,n)$

(d) Determine the truth value of : $\neg \forall n \ P(0,n)$. True or False.

(e) Write the following proposition in English . $\exists n \forall m \ P(m,n)$

(f) Determine the truth value of: $\exists n \forall m \ P(m,n)$. True or False.

(g) Write the following proposition in English . $\forall n \exists m \ P(m,n)$

(h) Determine the truth value of: $\forall n \exists m \ P(m,n)$. True or False.

11. Suppose the variable x represents students and y represents courses, and:
$M(y)$: y is a math course $F(y)$: x is a Freshman
$T(x,y)$: x is taking y

Write each of the following statements using the above predicates and any needed quantifiers:
(a) There is a course that every freshman is taking.

(b) There is a course that some freshman is taking.

(c) There is a unique course that every freshman is taking.

12. Let W denote the whole numbers {1, 2, 3, …}, N = {0, 1, 2, 3, …} and Z = {-3, -2, -1, 0, 1, 2, 3, …}.
(a) True or False: Assume that the universe of discourse is N. $\forall n \; n > 0$.

(b) True or False: Assume that the universe of discourse is Z. $\forall n \; n > 0$.

(c) True or False: Assume that the universe of discourse is Z. $\forall n \, \forall m \; n-m = m-n$.

(d) True or False: Assume that the universe of discourse is Z.
$\forall n \, \forall m \; n+m = m+n$.

(e) True or False: Assume that the universe of discourse is W. $\exists n \, \exists m \; n-m = m-n$.

(f) True or False: Assume that the universe of discourse is N. $\exists n \, \exists m \; n-m = m-n$.

(g) True or False: Assume that the universe of discourse is Z. $\forall n \, \exists m \; nm = 1$.

(h) True or False: Assume that the universe of discourse is Z. $\forall n \, \exists m \; n-m = 1$.

13. Prove the logical equivalence $\neg\forall x \; (x \in A \rightarrow x \in B) \equiv \exists x \; (x \in A \wedge x \notin B)$ using propositional equivalence, the laws of logic, and the negations of quantifiers, (give reasons at each step.)