Ch. 1.1 Logic

Def. A **Proposition** is a statement that is either true or false.

Example 1: Which of the following are propositions?

Statement	Proposition (yes or no)	If yes, then determine if it is true or false.
UHD is a University		
$1 + 3 = 0$		
$2 + 3$		
$x + 7 = 18$		
$x+7=18$ for $x = 9$		
$x+7=18$ for every real number x		

Notation: We use lower case letters to denote propositions.

Note: The area of logic that deals with propositions is called the **propositional calculus** or **propositional logic**.

Def. **Compound propositions** are new propositions formed from existing propositions using *logical operators*.

Note: We will discuss the following *logical operators* (also called connectives)

- Negation operator, $\neg$ (Other books use $\sim$)\
- Conjunction operator, $\wedge$
- Disjunction operator, $\vee$
- Exclusive or, $\oplus$
- Implication, $\rightarrow$
- Biconditional, $\leftrightarrow$

Def. Let p be a proposition. The statement " it is not the case that p" is a proposition formed from p and the negation operator, called **the negation of *p***, which we denote by $\neg p$. This proposition is read as "not p". The truth-value of $\neg p$ is true when p is false and false when p is true.

Truth Table

Example 2: Let p be the statement "*Bill Clinton was the president of the U.S.A in 2006*".

a. Write the statement $\neg p$ as an English statement.

b. What is the truth-value of $\neg p$?

Definition. Let p and q be propositions. The statement "p and q" is a proposition formed from p, q and the conjunction operator, called **the conjunction of *p* and *q***, which we denote by $p \wedge q$. The truth-value of $p \wedge q$ is true when both p and q are true, otherwise $p \wedge q$ is false.

Truth Table

Def. Let p and q be propositions. The statement "p or q" is a proposition formed from p, q and the disjunction operator, called **the disjunction of *p* and *q***, which we denote by $p \vee q$. The truth-value of $p \vee q$ is false when both p and q are false, otherwise $p \vee q$ is true.

Truth Table

Example 3: Let p be the statement "*Houston is the capital of Texas*" and let q be the statement "*Houston is a city in Texas*". Determine the following:

a. The truth-value for $p \wedge q$.

b. The truth-value for $p \vee q$.

c. The truth value for $\neg p \wedge q$

d. The truth value for $\neg (p \vee q)$

e. The truth value for $\neg p \vee q$

Def. Let p and q be propositions. The statement "p or q exclusively" is a proposition formed from p, q and the exclusive or operator, called the **exclusive or of p and q**, which we denote by $p \oplus q$. The truth-value of $p \oplus q$ is true when exactly one of p or q is true, otherwise $p \oplus q$ is false.

Truth Table

Comparison: A familiar example of the use of an "exclusive or" is a restaurant menu. "Price of the entree includes soup or salad". Note that one or the other is included in the price *but not both*. A familiar example of the use of inclusive or is the prerequisite for this course. "Students who have taken calculus or computer science, can take this course." Note that if a student has taken both courses, then the student may still take this course.

Definition. An **implication** (or **conditional statement**) is statement of the form "if p then q", denoted $p \rightarrow q$. Such propositions are also read as "p implies q". The "if part" is called the **hypothesis** (or **premise**), and the "then part" is called the **conclusion** (or **consequence**). The implication if p then q is false only when the hypothesis is true and the conclusion is false.

p	q	$p \rightarrow q$
T	T	
T	F	
F	T	
F	F	

Terminology: An implication $p \rightarrow q$ may be worded in several different ways:

- If p, then q
- If p, q
- q if p
- p only if q
- p is sufficient for q
- q is necessary for p

Examples 4:

a. True or False : If *2* is an even integer, then today is Friday.

b. Write the statement in the form "if p, then q" and determine the truth-value. *1 + 1 =2 if 3 + 3 =7.*

c. Write the statement in the form "if p, then q" and determine the truth-value. *1 + 1 = 2 only if 3 + 3 = 7.*

Definitions. Given the implication $p \rightarrow q$:

$q \rightarrow p$ is the **converse** of the implication $p \rightarrow q$.
$\neg p \rightarrow \neg q$ is the **inverse** of implication $p \rightarrow q$.
$\neg q \rightarrow \neg p$ is the **contrapositive** of implication $p \rightarrow q$.

Exercise 5: Write the converse, the inverse, and the contrapositive of the given implication:

If *cows eat grass*, then *2 + 3 = 4.*	Proposition	True or False?
	Converse	True or False?
	Inverse	True or False?
	Contrapositive	True or False?

Exercise 6: True or False : An implication and its converse have the same truth value.

Definition. The **biconditional proposition** denoted by $p \leftrightarrow q$ is the conjunction of $p \rightarrow q$ and $q \rightarrow p$. The proposition $p \leftrightarrow q$ is read as p if and only if q. The biconditional proposition is true when both p and q have the same truth-values and is false otherwise.

p	q	$p \leftrightarrow q$
T	T	
T	F	
F	T	
F	F	

Example 7: Determine the truth-values for the following:

a. The number *2* is an odd integer if and only if today is Friday. ____
b. If *3 + 3 = 7*, *1 + 1 = 2* and conversely if *1 + 1 = 2, 3 + 3 = 7*. ____

Exercise 8. Let *p* and *q* be the propositions

p: It is below freezing.
q: It is snowing

Write the following propositions using the symbols *p* and *q* and any appropriate logical connectives.

Use the following (a) –(d) to fill in the blank for i) and ii)
(a) $p \wedge q$ (b) $p \vee q$ (c) $p \rightarrow q$ (d) $q \rightarrow p$ (e) none of these

i) It is below freezing and snowing. _____
ii) It is below freezing if it is snowing _____

Use the following (a) –(d) to fill in the blank for iii) - vi)
(a) $p \oplus q$ (b) $p \vee q$ (c) $p \rightarrow q$ (d) none of these

i) It is not below freezing and it is not snowing. _____
ii) It is either below freezing or snowing (or both). _____
iii) It is either below freezing or snowing (but not both). _____
iv) It is below freezing if and only if it is snowing. _____

Exercise Construct a truth table for each of the following compound propositions. Note: your truth-table will require 2^n rows, where *n* is the number of simple propositions in the compound proposition.
a) $(p \wedge q) \vee \neg q$

b) $(p \vee q) \wedge \neg r$

An Application Computer circuitry

- ## Logic and Bit operations

Today's computers would have been impossible without an entirely original theory of logic developed by the English mathematician, George Boole. In 1854 he laid the groundwork for modern information theory in "An Investigation of the Laws of Thought," reducing logic to a simple algebra in which "reasoning" is carried out by manipulating formulas. His theory of logic recognized three basic operations -- AND, OR, and NOT. Then, in 1937, C. E. Shannon, an MIT graduate student, showed that Boolean symbolic logic could be applied to the analysis of switching circuitry and showed how logical algebra could be performed by relays.

Def. A **bit string** is a sequence of zero or more bits. The **length of the string** is the number of bits in the string.

Computer **bit operations** correspond to the logical connectives in the following way:

Let 1 represent *true* and 0 represent *false*. Further let x and y be bits.

x	y	x *OR* y $(x \vee y)$	x *AND* y $(x \wedge y)$	x X*OR* y $(x \oplus y)$

We can extend bit operations to bit strings of the same length. The operations are then called bitwise OR, bitwise AND and bitwise XOR.

Exercise: Determine the bitwise OR, the bitwise AND and bitwise XOR of the following strings 01 1011 1110 and 11 0001 1011