

Ch 5.1 The Basics of Counting

THE INCLUSION-EXCLUSION PRINCIPLE (SUM RULE):

If a first task can be done in n_1 ways and second task in n_2 ways, and if these tasks **can** be done at the same time, then there are $n_1 + n_2 - b$ ways to do either task. The number b denotes the number of ways to do both tasks.

$$*** |A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2|$$

Note. The inclusion-exclusion principle can be extended to more than two tasks.

THE PRODUCT RULE: Suppose that a procedure can be broken down into two tasks. If there are n_1 ways to do the first task and n_2 ways to do the second task after the first task has been done, then there are $n_1 n_2$ ways to do the procedure.

Note. The product rule can be extended to more than two tasks.

EXAMPLES:

1. Suppose that either a member of the mathematics faculty or a student who is a mathematics major is chosen as a representative to a university committee. How many different choices are there for this representative if there are 37 members of the mathematics faculty and 83 mathematics majors?

2. How many different bit strings are there of length seven?

3. Exercise. How many bit strings of length 10 begin and end with a 1?

4. How many functions are there from a set with m elements to one with n elements?

5. How many one-to-one functions are there from a set with m elements to one with n elements?

6. Use the product rule to show that the number of different subsets of a finite set S is $2^{|S|}$.

7. Exercise. How many positive integers not exceeding 1000 are
 - a) divisible by 7?
 - b) divisible by both 7 and 11?
 - c) divisible by 7 but not by 11?
 - d) divisible by exactly one of 7 and 11?
 - e) divisible by neither 7 nor 11?
 - f) have distinct digits?