Ch. 1.4
Nested Quantifiers

Main idea: Quantifiers may appear within the scope of other quantifiers.

Example 1: Assume that the universe of discourse for the variables x and y consists of all real numbers. The statement $\forall x \forall y(x+y=y+x)$ says that for every real number x and for every real number y, $x + y = y + x$. This is the _____________ law for addition of real numbers.

Example 2: Assume that the universe of discourse for the variables x and y consists of all real numbers. The statement $\forall x \exists y(x+y=0)$ says that for every real number x there exists a real number y, such that $x + y = 0$. This is the _____________ law for addition of real numbers.

Example 3: Assume that the universe of discourse for the variables x and y consists of all real numbers.

i. Translate the statement $\exists x \forall y(x+y=0)$ to English.

ii. What is the truth value of $\exists x \forall y(x+y=0)$.

The **order of quantifiers** of is important unless all the quantifiers are universal of all existential. As we have seen in the previous two examples $\exists x \forall y P(x,y)$ and $\forall x \exists y P(x,y)$ are not logically equivalent.

$\forall x \exists y P(x,y)$

$\exists x \forall y P(x,y)$

Example 4: Translate the statement "The sum of two positive integers is positive" into a logical expression.

Example 5: Translate the statement "Every real number except zero has a multiplicative inverse" into a logical expression.

Example 6: Assume the universe of discourse is the set of elements a set U. Let A and B be subsets of U. A function from A to B is *one-to-one* if for every a and b in A, $a=b$ whenever $f(a) = f(b)$.

i) Express the definition in terms of logical connectives and quantifiers.

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ii) Using part i) explain when a function is not *one-to-one*.

Example 7: Determine the truth value of each of the following statements if the universe of discourse for all variables consists of all integers.

a) $\forall n \exists m(n^2 < m)$

b) $\exists n \forall m(n^2 < m)$

c) $\neg \exists n \forall m(n^2 < m)$

d) $\exists n \forall m(nm = m)$

e) $\exists n \forall m(nm = m)$

f) $\exists n \exists m(n^2 + m^2 = 5)$

g) $\exists n \exists m(n^2 + m^2 = 6)$

f) $\exists n \exists m(n + m = 4 \wedge n - m = 1)$

h) $\exists n \exists m(n + m = 4 \wedge n - m = 2)$

i) $\forall n \forall m \neg(n + m = 4 \wedge n - m = 2)$

j) $\forall n \forall m(n + m \neq 4 \vee n - m \neq 2)$