

1. Consider the function $g(x) = 8 - 12x + 6x^2 - x^3$ that has the rate of change functions $g'(x) = -12 + 12x - 3x^2$ and $g''(x) = 12 - 6x$. The function g has a possible turning point and a possible inflection point located at the coordinates $(2, 0)$. Answer the following questions.

1a. Evaluate $g'(0) =$ Does this result imply that the function g is increasing or decreasing at the input $x = 0$?

1b. Evaluate $g'(3) =$ Does this result imply that the function g is increasing or decreasing at the input $x = 3$?

1c. Use your results from parts a–b to answer the following question: Is $(2, 0)$ an actual turning point on the graph of g ? Explain.

1d. Evaluate $g''(0) =$ Does this result imply that the function g is concave up or concave down at the input $x = 0$?

1e. Evaluate $g''(3) =$ Does this result imply that the function g is concave up or concave down at the input $x = 3$?

1f. Use your results from parts d–e to answer the following question: Is $(2, 0)$ an actual inflection point on the graph of g ? Explain.

2. A function is shown below along with its two rate of change functions. Find: i) the y -intercept; ii) the x -intercept(s); iii) the *coordinates* of possible turning points; iv) the *coordinates* of possible inflection points. Then use this information to suggest a graphing window for the function.

$$f(x) = x^3 - 3x^2 ; f'(x) = 3x^2 - 6x ; f''(x) = 6x - 6$$

i) y -intercept

ii) x -intercept(s)

iii) coordinates of possible turning points

iv) coordinates of possible inflection points

v) graphing window [____, ____] \times [____, ____]

3. Consider the function $f(x) = (x - 1)^7$ that has the rate of change functions $f'(x) = 7(x - 1)^6$ and $f''(x) = 42(x - 1)^5$. The function f has a possible inflection point located at the coordinates $(1, 0)$. Answer the following questions.

3a. Evaluate $f''(0) =$

- i. -1
- ii. 7
- iii. -42
- iv. 42

Does this result imply that the function f is concave up or concave down at the input $x = 0$?

- i. concave up
- ii. concave down
- iii. neither

Consider the function $f(x) = (x - 1)^7$ that has the rate of change functions $f'(x) = 7(x - 1)^6$ and $f''(x) = 42(x - 1)^5$.

3b. Evaluate $f''(2) =$

- i. 1
- ii. 7
- iii. 42
- iv. -42

Does this result imply that the function f is concave up or concave down at the input

$x = 2$?

- i. concave up
- ii. concave down
- iii. neither

3c. Use your results from parts a–b to answer the following question: Is $(1, 0)$ an actual inflection point on the graph of f ? Explain.

- i. Yes, the function changes from concave up to concave down at $x = 1$
- ii. Yes, the function changes from concave down to concave up at $x = 1$
- iii. No, the function does not change concavity at $x = 1$

4. A function is shown below along with its two rate of change functions. Find: i) the y -intercept; ii) the x -intercept(s); iii) the *coordinates* of possible turning points; iv) the *coordinates* of possible inflection points. Then use this information to suggest a graphing window for the function.

$$f(x) = x^4 - 8x^3 \quad ; \quad f'(x) = 4x^3 - 24x^2 \quad ; \quad f''(x) = 12x^2 - 48x$$

i) y -intercept

ii) x -intercept(s)

iii) coordinates of possible turning points

iv) coordinates of possible inflection points

v) graphing window [_____, _____] × [_____, _____]

5. An economy's CPI (Consumer Price Index) is given by

$$I(t) = -0.2t^3 + 3t^2 + 100$$

where $t = 0$ corresponds to 1991.

Calculate and compare $I'(6)$ and $I''(6)$ to show that even though the CPI was rising at the beginning of 1997, “inflation was moderating” at that time.

(Inflation is the rate at which prices are increasing.)