MATH 1306	***Spring 2007***	***Name*** ______________________
Homework Handout # 7		***Student ID*** ______________

1. In each part below, a function is given along with its corresponding rate of change function. Find: i) the slope of the tangent line at the input 0, and; ii) the <u>coordinates</u> of all possible turning points.

1a. $f(x) = x^2 - 6x - 7$; $f'(x) = 2x - 6$

i) the slope of the tangent line at the input 0 [ ]

ii) coordinates of all possible turning points [ ]

1b. $g(x) = x^3 + 4x^2 - 3x$; $g'(x) = 3x^2 + 8x - 3$

i) the slope of the tangent line at the input 0 [ ]

ii) coordinates of all possible turning points [ ]

2. In each part below, a function is given along with its corresponding rate of change function. For each of these functions, find: i) the y-intercept; ii) the x-intercept(s); iii) the slope of the tangent line at the input 0; iv) the coordinates of possible turning points. Then use this information to suggest a graphing window for the function. (Note: You answered parts i and ii in Homework Handout # 1.)

2a. $f(x) = -5x + 10$; $f'(x) = -5$

i) y-intercept [ ]

ii) x-intercept(s) [ ]

iii) the slope of the tangent line at the input 0 [ ]

iv) coordinates of all possible turning points [   ]

v) graphing window $[\underset{x\,Min}{\underline{\hspace{2cm}}}, \underset{x\,Max}{\underline{\hspace{2cm}}}] \times [\underset{y\,Min}{\underline{\hspace{2cm}}}, \underset{y\,Max}{\underline{\hspace{2cm}}}]$

2b. $y = x^2 - 6x + 11$; $\frac{dy}{dx} = 2x - 6$

i) y-intercept [   ]

ii) x-intercept(s) [   ]

iii) the slope of the tangent line at the input 0 [   ]

iv) coordinates of all possible turning points [   ]

v) graphing window [_____, _____] × [_____, _____]

2c. $h(x) = x^4 + 4x^3$; $h'(x) = 4x^3 + 12x^2$

i) y-intercept [   ]

ii) x-intercept(s) [   ]

iii) the slope of the tangent line at the input 0 [   ]

iv) coordinates of all possible turning points [   ]

v) graphing window [_____, _____] × [_____, _____]

3. Consider the function $g(x) = 2x^3 - 3x^2 - 12x$ that has the rate of change function $g'(x) = 6x^2 - 6x - 12$. The function g has a turning point located at the coordinates $(2, -20)$. Answer the following questions.

3a. Evaluate $g'(1) =$ ☐ Does this result imply that the function g is increasing or decreasing at the input $x = 1$? ☐

3b. Evaluate $g'(3) =$ ☐ Does this result imply that the function g is increasing or decreasing at the input $x = 3$? ☐

3c. Use your results from parts a–b to answer the following question: Is the turning point $(2, -20)$ a peak on the graph of g, or a valley? Explain.

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4. In each part below, a function is given along with its two rate of change functions. For each of these functions, find: i) the y-intercept; ii) the x-intercept(s); iii) the coordinates of possible turning points; iv) the coordinates of possible inflection points. Then use this information to suggest a graphing window for the function. (Note: You answered parts i and ii in Homework Handout # 1.)

4a. $f(x) = 8x^2 - 2x^4$; $f'(x) = 16x - 8x^3$; $f''(x) = 16 - 24x^2$

i) y-intercept ☐

ii) x-intercept(s) ☐

iii) coordinates possible turning points ☐

iv) coordinates of all possible inflection points ☐

v) graphing window $[\_\_\_\_, \_\_\_\_] \times [\_\_\_\_, \_\_\_\_]$

4b. $y = 3x^{4/3} - 3x$; $\frac{dy}{dx} = 4x^{1/3} - 3$; $\frac{d^2y}{dx^2} = \frac{4}{3}x^{-2/3}$

i) y-intercept [ ]

ii) x-intercept(s) [ ]

iii) coordinates of all possible turning points [ ]

iv) coordinates of all possible inflection points [ ]

v) graphing window $[\_\_\_\_, \_\_\_\_] \times [\_\_\_\_, \_\_\_\_]$

5. A function is shown below along with its two rate of change functions. Find: i) the y-intercept; ii) the x-intercept(s); iii) the coordinates of possible turning points; iv) the coordinates of possible inflection points. Then use this information to suggest a graphing window for the function. (Note: You answered parts i and ii in Homework Handout # 1.)

$f(x) = (x-1)(x+1)(x-2)$; $f'(x) = 3x^2 - 4x - 1$; $f''(x) = 6x - 4$

i) y-intercept [ ]

ii) x-intercept(s) [ ]

iii) coordinates of all possible turning points [ ]

iv) possible inflection points [ ]

v) graphing window $[\_\_\_\_, \_\_\_\_] \times [\_\_\_\_, \_\_\_\_]$

6. Given below is a function along with its corresponding rate of change function. Find: i) the y-intercept; ii) the x-intercept(s); iii) the slope of the tangent line at the input 0; iv) the <u>coordinates</u> of possible turning points. Then use this information to suggest a graphing window for the function. (Note: You answered parts i and ii in Homework Handout # 1.)

6a. $g(x) = x^2 + 2x - 15$; $g'(x) = 2x + 2$

i) y-intercept ☐

ii) x-intercept(s) ☐

iii) the slope of the tangent line at the input 0 ☐

iv) coordinates of all possible turning points ☐

v) graphing window $[\_\_\_\_, \_\_\_\_] \times [\_\_\_\_, \_\_\_\_]$

7. In each part below, a function is given along with its corresponding rate of change function. Find: i) the y-intercept; ii) the x-intercept(s); iii) the slope of the tangent line at the input 0; iv) the <u>coordinates</u> of possible turning points. Then use this information to suggest a graphing window for the function.

7a. $f(x) = x^2 - 4x - 21$; $f'(x) = 2x - 4$

i) y-intercept ☐

ii) x-intercept(s) ☐

iii) the slope of the tangent line at the input 0 ☐

iv) coordinates of all possible turning points [ ]

v) graphing window $[\_\_\_\_\_, \_\_\_\_\_] \times [\_\_\_\_\_, \_\_\_\_\_]$

7b. $g(x) = x^3 - 12x$; $g'(x) = 3x^2 - 12$

i) the slope of the tangent line at the input 0 [ ]

ii) coordinates of all possible turning points [ ]

iii) graphing window $[\_\_\_\_\_, \_\_\_\_\_] \times [\_\_\_\_\_, \_\_\_\_\_]$

8. Consider the function $g(x) = x^3 - 6x^2 + 12x - 8$ that has the rate of change function $g'(x) = 3x^2 - 12x + 12$. The function g has a possible turning point located at the coordinates $(2, 0)$. Answer the following questions.

8a. Evaluate $g'(0) =$ [ ] Does this result imply that the function g is increasing or decreasing at the input $x = 0$? [ ]

8b. Evaluate $g'(3) =$ [ ] Does this result imply that the function g is increasing or decreasing at the input $x = 3$? [ ]

8c. Use your results from parts a–b to answer the following question: Is $(2, 0)$ an actual turning point on the graph of g? Explain.

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Answers To Odd-numbered Problems

1a. i) slope of tangent line at input 0 = $f'(0) = -6$
ii) coordinates of possible turning points are $(x, y) = (3, -16)$
Justification:
To find x: solve $f'(x) = 0$ or $2x-6 = 0$. You get $x=3$.
To find y: $y = f(3) = (3)^2 - 6(3) - 7 = -16$

1b. i) slope of tangent line at input 0 = $g'(0) = -3$
ii) coordinates of possible turning points are $(-3, 18)$ and $(1/3, -14/27) \approx (0.3, -0.5)$
Justification:
To find x: solve $f'(x) = 0$ or $3x^2+8x-3 = 0$. You get $x=-3$ and $x=1/3$.
To find y: $y = f(-3) = (-3)^3 +4(-3)^2 - 3(-3) = 18$
And $y = f(1/3) = (1/3)^3 +4(1/3)^2 - 3(1/3) = -14/27$

3a. $g'(1) = -12$ and so the function g is decreasing at the input $x = 1$
3b. $g'(3) = 24$ and so the function g is increasing at the input $x = 3$
3c. The turning point $(2, -20)$ is a valley (local minimum) on the graph of g since the function changes from decreasing to increasing at $x=2$.

5. i) y-intercept = $f(0) = 2$
ii) x-intercepts are $x = -1$, $x = 1$, $x = 2$
iii) coordinates of the possible turning points are approximately $(-0.2, 2.1)$ and $(1.5, -0.6)$
Hint: Use the quadratic formula $x = [\ -b \pm \sqrt{(b^2 - 4ac)}\ ]/2a$
iv) coordinates of the possible inflection point are approximately $(2/3, 0.7) \approx (0.7, 0.7)$
v) a possible graphing window is [xMin, xMax] x [yMin, yMax] = [-2, 3] x [-1, 3]
Note: Different persons may answer differently.

7a. i) y-intercept = $f(0) = -21$
ii) x-intercepts are $x = -3$, $x = 7$
iii) slope of tangent line at input 0 = $f'(0) = -4$
iv) coordinates of possible turning points are $(2, -25)$
v) a possible graphing window is [-4, 8] x [-17, 17]

7b. i) slope of tangent line at input 0 = $g'(0) = -12$
ii) coordinates of possible turning points are $(-2, 16)$ and $(2, -16)$
iii) a possible graphing window is [-4, 4] x [-17, 17]