MATH 1306	*Spring 2007*	*Name*____________________________	
Homework Handout # 6		***Student ID*** ____________________	

1. Given the function $f(x) = \dfrac{x - 4}{5 - x}$:

Let's use a table of average rates of change to estimate the instantaneous rate of change of f at $x = 4$.

1a. Compute the missing average rate of change in the table.

increment h	average rate of change of f between 4 and $4 + h$
	$f_{[4\,,\,4+0.5]} =$ __________
	$f_{[4\,,\,4+0.25]} = 1.3333$
	$f_{[4\,,\,4+0.1]} = 1.1111$
	$f_{[4\,,\,4+0.01]} = 1.0101$
	$f_{[4\,,4+0.001]} = 1.0010$

Study the patterns in the table:

i) From row to row, in each average rate of change, one of the input values is fixed;
What is its value? $x =$ [ ]

ii) For each average rate of change $f_{[x\,,\,x+h]}$ in the table, what is the value of the increment? Write the answers in the first column of the table.

iii) Complete the following statement.

As the increment h approaches the number _____, the average rate of change values $f_{[4,4+h]}$ approach the number _____ .

Therefore, $f\,'(4) \approx$ [ ]

1b. What is the slope of the tangent line to the graph of f at the input $x = 4$? [ ]
Explain.

[ ]

2. Given the function $f(x) = \dfrac{x^2 - 2x}{x - 5}$:

Let's use a table of average rates of change to estimate the instantaneous rate of change of f at $x = 0$.

2a. Compute the missing average rate of change in the table.

increment h	average rate of change of f between 0 and $0 + h$
	$f_{[0\,,\,0+1]} =$ ________
	$f_{[0\,,\,0+0.5]} = 0.3333$
	$f_{[0\,,\,0+0.1]} = 0.3877$
	$f_{[0\,,\,0+0.01]} = 0.3987$
	$f_{[0\,,\,0+0.001]} = 0.3998$

2b. Use the table to complete the statement.

As the increment h approaches the number ____, the average rate of change values $f_{[0,0+h]}$ approach the number ____

Therefore, $f\,'(0) \approx$ [ ]

2c. What is the slope of the tangent line to the graph of f at the input $x = 0$? [ ]
Explain.

[ ]

3. Given the function $f(x) = \dfrac{x^2 - 2x}{x - 5}$:

3a. Complete the following table.

$f_{[3\,,\,3+0.5]} =$ ________
$f_{[3\,,3+0.1]} = -2.9473$
$f_{[3\,,3+0.01]} = -2.7688$
$f_{[3\,,\,3+0.001]} = -2.7518$
$f_{[3\,,\,3+0.0001]} = -2.7501$

3b. Complete the statement.

As the increment h approaches the number _____, the average rate of change values $f_{[x,x+h]} = f_{[3,3+h]}$ approach the number _____

Therefore, $f\,'($______$) \approx$ [ ]

3c. What is the slope of the tangent line to the graph of f at the input $x = 3$? [ ]
Explain.

[ ]

4. This problem continues on to the next page.
Given the function $f(x) = \dfrac{3x - x^2}{x - 2}$:

4a. Complete the following table.

$f_{[0\,,\,0+1]} =$ ________
$f_{[0\,,\,0+0.5]} = -1.6667$
$f_{[0\,,\,0+0.1]} = -1.5263$
$f_{[0\,,\,0+0.01]} = -1.5025$
$f_{[0\,,\,0+0.001]} = -1.5003$

4b. Use the table to estimate $f'(0) \approx$ [ ]

4c. What is the slope of the tangent line to the graph of f at the input $x = 0$? [ ]
Explain.

[ ]

5. Given the function $f(x) = \dfrac{5x - x^2}{x - 2}$:

5a. Complete the following table.

$f_{[0,\,0+1]} = $ ________
$f_{[0,\,0+0.5]} = -3$
$f_{[0,\,0+0.1]} = -2.5789$
$f_{[0,\,0+0.01]} = -2.5075$
$f_{[0,\,0+0.001]} = -2.5007$

5b. Use the table to estimate $f'(0) \approx$ [ ]

5c. What is the slope of the tangent line to the graph of f at the input $x = 0$? [ ]
Explain.

[ ]

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Answers To Selected Odd-numbered Problems

1a. $f_{[4,4+0.5]} = 1/0.5 = 2$

i) $x = 4$

ii) The values of the increment h are: 0.5, 0.25, 0.1, 0.01, 0.001

iii) As the increment h approaches the number 0, the average rate of change values $f_{[4,4+h]}$ approach the number 1.

So $f'(4) \approx 1$

1b. The slope of the tangent line at x equals the rate of change of f at x, $f'(x)$.
So the slope of the tangent line at $x = 4$ is approximately 1 because $f'(4) \approx 1$.

3a. $f_{[3,3+0.5]} = -4$

3b. As the increment h approaches 0, the average rate of change values $f_{[3,3+h]}$ approach the number –2.75.

So $f'(3) \approx -2.75$

3c. The slope of the tangent line at $x = 3$ is approximately –2.75 since the slope of the tangent line equals $f'(3)$.

5a. $f_{[0,0+1]} = -4$

5b. $f'(0) \approx -2.5$

This is because as the increment h approaches 0, the average rate of change values $f_{[0,0+h]}$ approach the number –2.5.

5c. The slope of the tangent line at $x = 0$ is approximately –2.5; this is because the slope of the tangent line equals $f'(0)$.