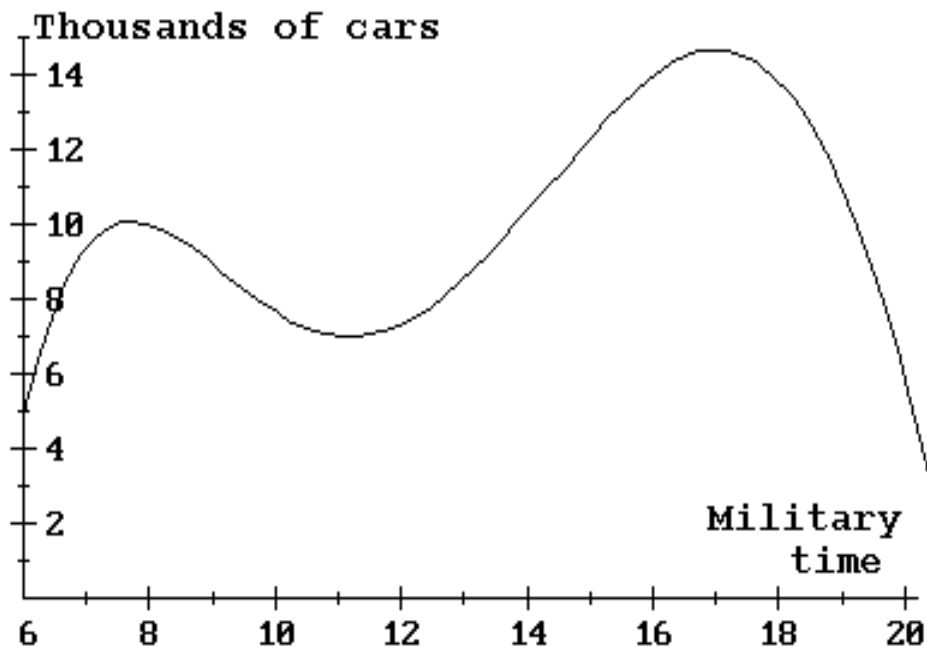


1. The following graph shows the average number of cars (in thousands) traveling on the Gridlock Expressway in Gotham City during a typical weekday from 6 am to 8 pm (the input axis is marked off in military time). For example, military time 14 corresponds to 2 pm. Suppose  $C$  is the number of cars (in thousands) on this expressway at time  $t$  (military time).



- 1a. Sketch the tangent line to the graph at the input  $t = 16$ .

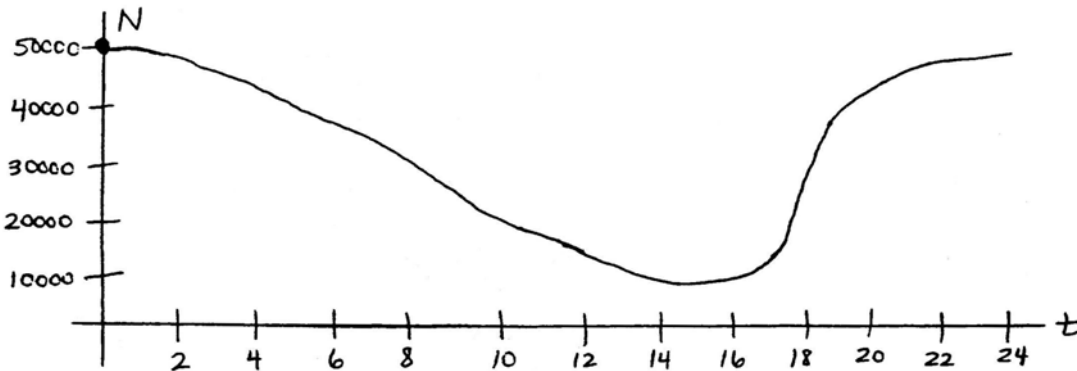
- 1b. Estimate the slope of the tangent line drawn in part 1a.  $m =$

- 1c. Estimate the instantaneous rate of change of  $C$  at the input  $t = 16$ .

- 1d. Interpret your answer from part 1c in the problem context using a complete sentence.

- 1e. Evaluate  $C'(17) =$

2. Victims of the wombat fever virus are treated with a drug that slows the replication of the virus in their bodies. Unfortunately, the virus develops resistance to this drug over time and the effect of the drug slowly wears off. The function  $N(t)$  graphed below outputs the typical number of virus particles circulating per milliliter of blood in a patient being treated for wombat fever. The input variable  $t$  represents the time (in months) after the patient begins treatment.



2a. Evaluate  $N(0)$  and interpret this value in the context of the problem.

- i. There are no virus particles circulating in the blood of a patient being treated for wombat fever.
- ii. When a patient begins treatment for wombat fever, there are typically 50 000 virus particles circulating per milliliter of blood.
- iii. A patient beginning treatment for wombat fever typically has 50 000 virus particles in their body.
- iv. The number of virus particles continuously decreases after beginning treatment for wombat fever.

2b. Sketch the tangent line to the graph of  $N$  at the input  $t = 20$ .

2c. Estimate the slope of the tangent line drawn in Part 2b.  $m =$

- i. 42 000
- ii. -3000
- iii. 3000
- iv. 1000

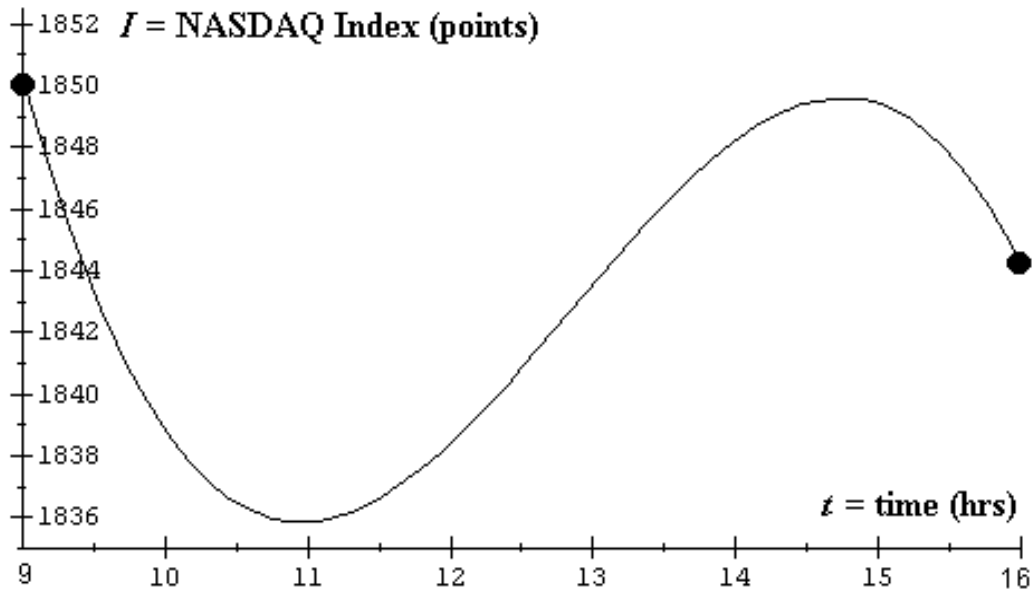
2d. Estimate  $N'(20) \approx$  . Explain how you got your estimate.

- i. 42 000
- ii. -3000
- iii. 3000
- iv. 1000

**2e.** Interpret your estimate from Part 2d in the context of the problem.

- i. After a patient has 20 hours of treatment for wombat fever, the number of virus particles circulating in the bloodstream is 42 000.
- ii. The number of virus particles circulating in the bloodstream is decreased by 3000.
- iii. After a patient has 20 hours of treatment for wombat fever, the number of virus particles circulating in the bloodstream is increasing by 3000 per hour.
- iv. After 1000 hours of treatment, there are 20 virus particles circulating in the bloodstream of the patient.

**3.** The NASDAQ Composite Index is a numerical value that roughly measures the price of all stocks traded on the NASDAQ Stock Exchange. This exchange has become more important in recent years, since the stocks of many high-tech companies are traded there. The following graph depicts the value of the NASDAQ index ( $I$ ) over the course of a recent trading day. The input axis is marked off in military time, where  $t = 9$  corresponds to 9 am and  $t = 16$  corresponds to 4 pm.



**3a.** Sketch the tangent line to the graph at the input  $t = 14$ .

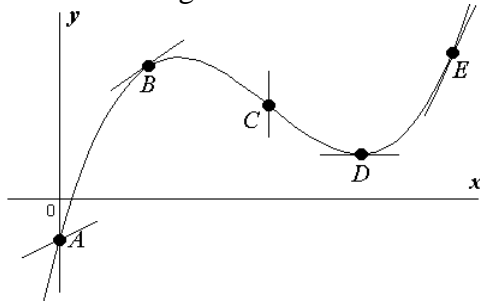
**3b.** Estimate the slope of the tangent line drawn in part 3a.  $m =$

**3c.** Estimate the instantaneous rate of change of  $I$  at the input  $t = 14$ .

**3d.** Interpret your answer from part 3c in the problem context using a complete sentence.

**3e.** Evaluate  $I'(11) =$

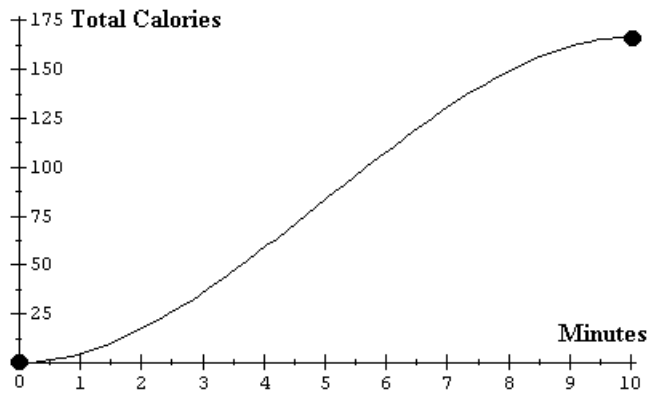
4. Five points labeled  $A$  through  $F$  have been marked along the function curve shown below. At each point, a portion of a line has been drawn that touches the curve at only the marked point. For each of these lines, determine if it is tangent to the curve at the corresponding point.



4a.  $A$  Yes  No   
 4c.  $C$  Yes  No   
 4e.  $E$  Yes  No

4b.  $B$  Yes  No   
 4d.  $D$  Yes  No

5. The Hard-Body Elliptical Crosstrainer fitness machine has an LCD display that graphs the total calories burned during a workout. Suppose the display appeared roughly as shown below after Imani's last workout on the machine. Let  $T$  be the total calories burned after  $m$  minutes of the workout.



5a. Sketch the tangent line to the graph at the input  $m = 8$ .

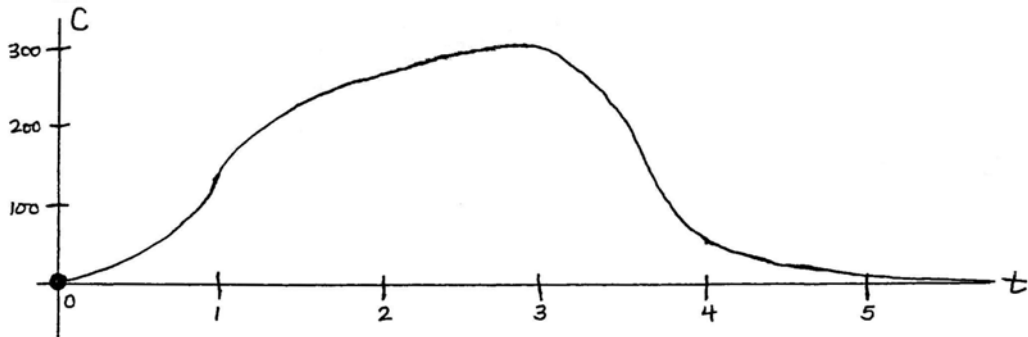
5b. Estimate the slope of the tangent line drawn in part 5a.  $slope =$

5c. Estimate the instantaneous rate of change of  $T$  at the input  $m = 8$ .

5d. Interpret your answer from part 5c in the problem context using a complete sentence.

5e. Evaluate  $T'(8) =$

6. The function  $C$  graphed below outputs the amount of aspirin circulating in the bloodstream of a person  $t$  hours after 350 mg of aspirin is consumed.



6a.

Evaluate  $C(1)$  and interpret this value in the context of the problem.

- i. There was 1 mg of aspirin in the person's bloodstream.
- ii. The person should take another 350 mg after 1 hour.
- iii. One hour after a person takes 350 mg of aspirin, there are 130 mg of aspirin circulating in their bloodstream.
- iv. After 1 hour, the person has 220 mg of aspirin circulating in the bloodstream.

6b. Sketch the tangent line to the graph of  $C$  at the input  $t = 2$ .

6c. Estimate the slope of the tangent line drawn in Part 6b.  $m \approx$

- i. 270
- ii. -80
- iii. 130
- iv. 60

6d. Estimate  $C'(2) \approx$  . Explain how you got your estimate.

- i. 270
- ii. -80
- iii. 130
- iv. 60

6e. Interpret your estimate from Part 6d in the context of the problem.

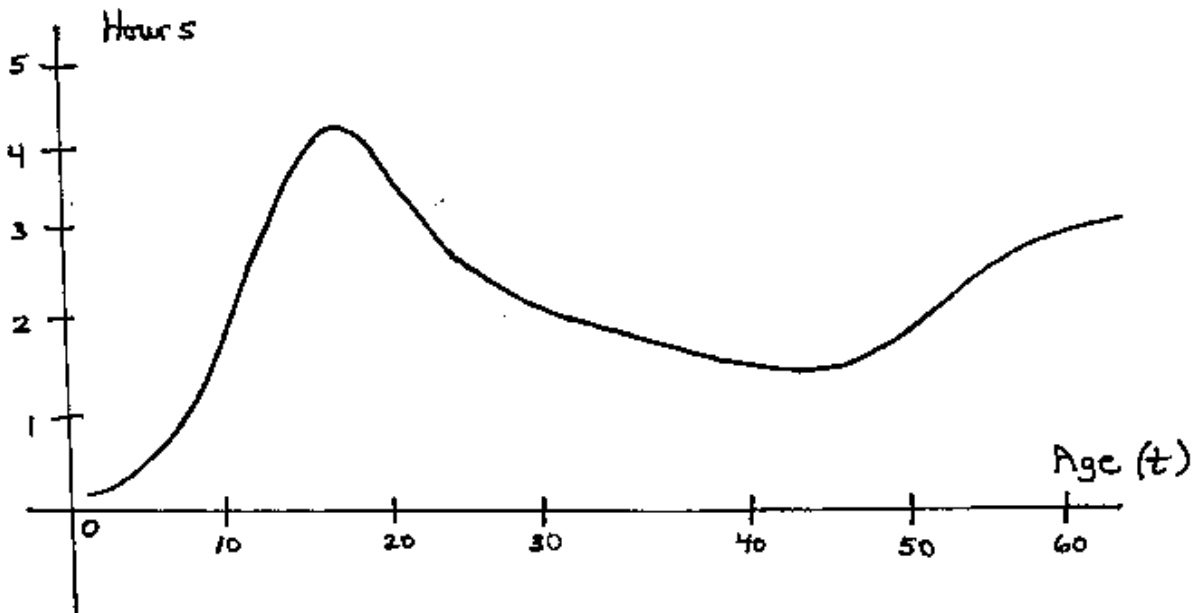
i. Two hours after taking 350 mg of aspirin, the amount of aspirin in the bloodstream is increasing by 60 mg per hour.

ii. Two hours after taking 350 mg of aspirin, the amount of aspirin in the bloodstream is increasing by 270 mg per hour.

iii. After taking 350 mg of aspirin, the amount of aspirin in the bloodstream is increasing by 60 mg every 2 hours.

iv. After 2 hours, the amount of aspirin in the bloodstream is 60 mg.

7. The function  $V$  graphed here depicts the number of hours each day typical Americans from age 3 years to age 60 years spend watching TV.



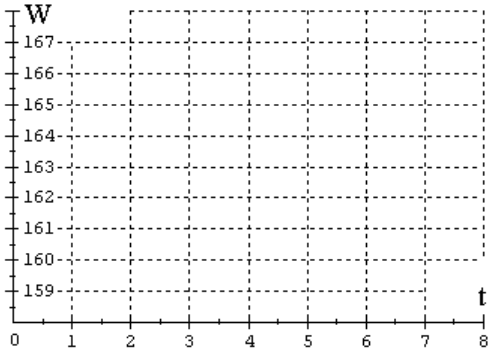
7a. Suppose you are given that  $V'(40) = -0.025$ . Interpret this value in the problem context using a complete sentence.

**8.** In 1990, the average weight of an American adult was 160 lbs. By 1998, the average weight had risen to 166 lbs. Let  $W(t)$  be the average weight of an American adult in year  $t$ , where  $t = 0$  corresponds to 1990.

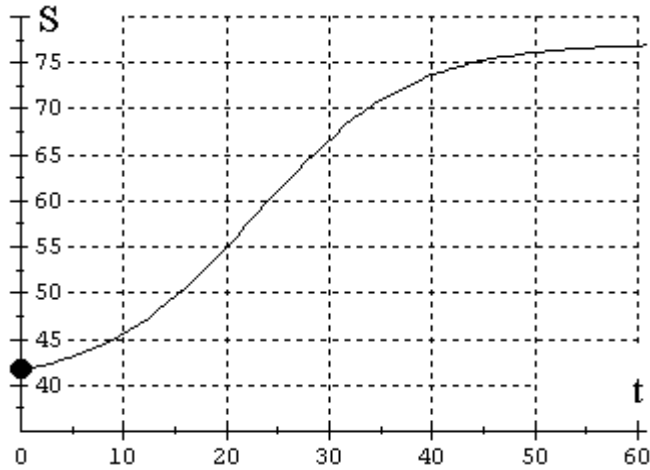
**8a.** Compute  $W_{[0,8]} =$   Interpret this value in the problem context using a complete sentence.

**8b.** Find the  $y$ -intercept of  $W$ .  Interpret this value in the problem context using a complete sentence.

**8c.** Suppose the average weight of an American adult increased at a steady rate between 1990 and 1998. Draw the graph of  $W$  on the following set of axes.



**9.** The function  $S$  graphed here outputs the number of millions of persons predicted to be receiving Social Security benefits  $t$  years after 1990.



**9a.** Evaluate  $S(0)$  and interpret this value in the context of the problem.

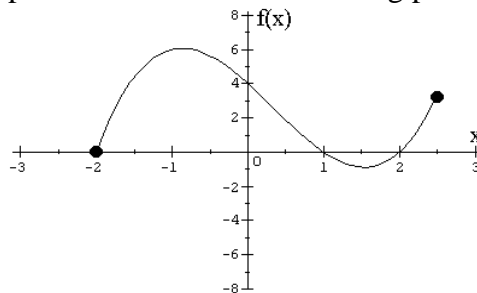
**9b.** Sketch the tangent line to the graph of  $S$  at the input  $t = 40$ .

**9c.** Estimate the slope of the tangent line drawn in Part 9b.  $m \approx$

**9d.** Estimate  $S'(40) \approx$  . Explain how you got your estimate.

**9e.** Interpret your estimate from Part 9d in the context of the problem.

**10.** A function named  $f$  is graphed below. Each of following parts pertains to this function.



Circle True or False for each of the following expressions.

True False **10a.**  $f(-1) > 0$

True False **10b.**  $f(3) > 0$

True False **10c.**  $f(0) = 4$

True False **10d.**  $f_{[0,1]} < 0$

True False **10e.**  $f_{[1,2]} > 0$

True False **10f.**  $f_{[-2,-1]} < 0$

True False **10g.**  $f'(2) = 0$

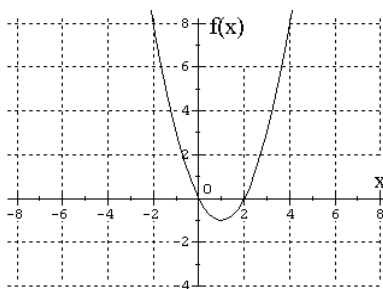
True False **10h.**  $f'(0) < 0$

**11.** Consider the function  $f(x) = x^2 - 2x$ .

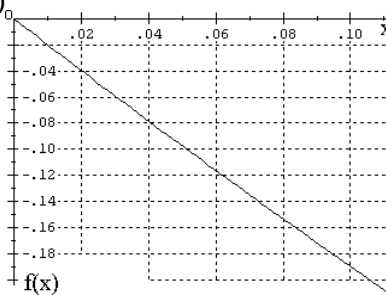
**11a.** Express the following value using function notation: The instantaneous rate of change of  $f$  at the input  $-1$ .

**11b.** Which of the following graphs i-iv of  $f$  can be used to accurately estimate the instantaneous rate of change of  $f$  at the input  $-1$ ?

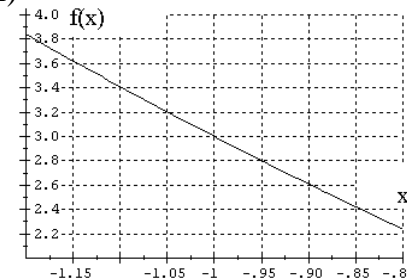
i)



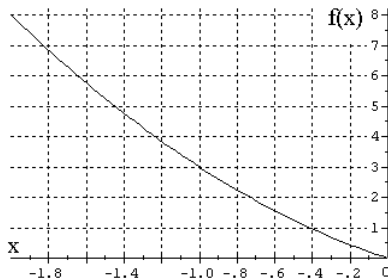
ii)



iii)



iv)



**11c.** Use your answer from part b to estimate the instantaneous rate of change of  $f$  at the input  $-1$ . *rate of change* =

**MATH 1306 – Handout # 4**  
**Answers To Selected Odd-numbered Problems**

- 1b. tangent line slope = 1.42 (Answers may vary)
- 1c. 1.42 is the instantaneous rate of change of  $C$  at  $t = 16$  since this is the slope of the tangent line at this point.
- 1d. At 4 pm ( $t=16$ ), the number of cars on the expressway is increasing by 1.42 thousand cars per hour, on a typical weekday.
- 1e.  $C'(17) = 0$  since the tangent line is a horizontal line and so has slope 0
- 
- 3b. tangent line slope = 3.5 (Answers may vary)
- 3c. 3.5 is the instantaneous rate of change of  $I$  at  $t=14$  since this is the slope of the tangent line at this point.
- 3d. At 2 pm ( $t=14$ ) on a recent trading day, the NASDAQ was increasing by 3.5 points per hour.
- 3e.  $I'(11) = 0$  since the tangent line at  $t = 11$  is a horizontal line and so has slope 0
- 
- 5b. tangent line slope = 16.3 (Answers may vary)
- 5c. 16.3 is the instantaneous rate of change of  $T$  at the input  $m = 8$
- 5d. After 8 minutes of working out on the cross trainer machine, the total calories burned by Imani is increasing by 16.3 calories per minute.
- 5e.  $T'(8) = 16.3$  since the tangent line at input 8 has slope 16.3 at this point
- 
- 7a. For a typical 40 year old American, the daily amount of time watching TV is decreasing by 0.025 hours per year. [0.025 hr = 1.5 min]
- 
- 9a.  $S(0) = 42$ ; In 1990, the number of persons receiving Social Security benefits was predicted to be 42 million.
- 9c. tangent line slope = 0.425 (Answers may vary)
- 9d.  $S'(40) = 0.425$ ; This is because  $S'(40)$  equals the slope of the tangent line at the input 40.
- 9e. In 2030 ( $t = 40$ ), the number of persons predicted to be receiving Social Security benefits is increasing by 0.425 million per year (or 425,000 per year).
- 
- 11a.  $f'(-1)$
- 11b. Graph iii, since it contains the point with input -1 and it is the smallest graphing window given
- 11c. rate of change = -4 since the slope of the tangent line is -4 at  $x = -1$  (Answers may vary)